# Liability Dollarization, Sudden Stops \& 

## Optimal Financial Policy

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#### Abstract

Banks in emerging markets intermediate capital inflows denominated in hard currencies (i.e. tradable goods) to fund loans denominated in domestic currency (i.e. domestic consumption units). This "liability dollarization" affects borrowing decisions via three effects absent from standard Sudden Stops models, in which domestic loans are in units of tradables. First, real depreciations reduce ex-post real interest rates, and hence the burden of repaying outstanding debt. Second, expected real appreciations reduce ex-ante real interest rates and increase the resources generated by issuing new debt. Third, the positive co-movement of consumption and real interest rates reduces the expected marginal cost of borrowing. These effects add an "intermediation externality" to the macroprudential externality of the standard models. Optimal policy under commitment is time-inconsistent, tightens access to debt when expectations of real appreciation rise, and does not require capital controls. In contrast, an optimal, time-consistent policy requires both domestic credit regulation and capital controls. Quantitatively, the model predicts higher debt ratios with milder Sudden Stops than the standard models, but it fits observed Sudden Stops better. The optimal policy is very effective but also complex, while simple rules optimized to maximize welfare are much less effective, and implemented with ad-hoc values can reduce welfare significantly. Welfare-improving policies favor taxing domestic debt more than capital inflows, and subsidizing inflows is part of the optimal policy.


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## 1 Introduction

Financial intermediation in emerging markets is characterized by what Calvo (2002) labeled "liability dollarization:" Banks intermediate capital inflows denominated in hard currencies (i.e. units of tradable goods) into domestic loans generally denominated in national currencies (i.e. units of national consumer prices). In South Korea or Mexico, for example, dollar inflows are lent out typically in domestic currency units, and the same happens also with Euro and Swiss Franc inflows in the emerging markets of Eastern Europe. A report by the Bank for International Settlements showed that in 2007, just before the global financial crisis, the ratio of foreign currency liabilities to total liabilities of commercial banks in emerging markets was about 40 percent in Latin America, 25 percent in Europe, and 15 percent in Asia, Africa and the Middle East, and the median ratio of external liabilities to gross loans in emerging markets was about 36 percent. ${ }^{1}$ Using IMF data, Eichengreen and Hausmann (1999) reported that in 1996, just before the Asian Crisis, the ratios of foreign liabilities to total assets in commercial banks ranged from 143 percent in Indonesia to 775 percent in Thailand.

This paper shows that taking into account the effects of liability dollarization on domestic borrowing decisions alters significantly the positive and normative implications of Sudden Stops models, which generally assume that domestic credit is denominated in units of tradables. In particular, optimal financial policy under commitment is time-inconsistent (i.e. lacks credibility) and does not justify the use of capital controls, as capital controls and regulation of domestic borrowing are shown to be equivalent. In contrast, the optimal time-consistent policy of a conditionally-efficient regulator (i.e. one that supports the pricing function of domestic debt in the decentralized equilibrium) requires a well-defined mix of domestic credit regulation and capital controls. In this case, capital controls sustain the credibility of the optimal policy.

The workhorse Sudden Stops (SS) model used to study macroprudential policy in emerging markets abstracts from liability dollarization, because it is built upon the canonical Dependent Economy framework of International Macroeconomics. ${ }^{2}$ This framework includes income and con-

[^0]sumption of tradable and nontradable goods but assumes that debt is denominated in units of tradables. Standard SS models consider a stochastic setup in which domestic agents borrow by selling non-state-contingent bonds denominated in units of tradables, and pledging as collateral a fraction of their total income, part of which originates in the nontradables sector. ${ }^{3}$ The key element of these models is that the collateral provided by the income from nontradables is valued at the market-determined relative price of nontradable goods relative to tradables, which yields two central implications: First, it introduces the Fisherian debt-deflation amplification mechanism, by which a binding collateral constraint triggers a feedback mechanism linking reduced borrowing capacity, decreased consumption of tradable goods, and collapsing relative prices. Second, it introduces a pecuniary externality, by which agents do not internalize in "good times" the effect of their borrowing decisions on relative prices and borrowing capacity in "bad times" when the collateral constraint binds. These two features of the SS setup are related, because the magnitude of the pecuniary externality is determined by the size of the Fisherian amplification effect on prices (see Mendoza (2016)). Quantitative studies (e.g. Bianchi (2011), Bianchi et al. (2016)) have shown that both financial amplification and pecuniary externalities are large in SS models, and that optimal macroprudential policy reduces significantly the frequency and magnitude of Sudden Stops.

The assumption that domestic debt is in units of tradables simplifies theoretical and quantitative work with SS models significantly, but it also rules out by construction liability dollarization. Intermediaries in these models can be viewed as either domestic or international banks that issue liabilities in tradables units at the world real interest rate, and lend them to domestic agents in the same units and at that same rate but requiring them to post collateral. ${ }^{4}$ Interestingly, other strands of the literature on emerging markets crises did introduce liability dollarization, particularly with the aim of studying aggregate implications of balance sheet effects and bank failures resulting from large currency devaluations (e.g. Choi and Cook (2004)).

To introduce liability dollarization in a model of Sudden Stops, capital inflows denominated in

[^1]units of tradables need to be distinguished from domestic debt denominated in units of the aggregate domestic price index. It is well-known that the denomination of the debt is not innocuous under perfect foresight and in stochastic models with incomplete markets, because of income effects that result from relative price movements, but the implications of liability dollarization for the Fisherian deflation mechanism, the nature of pecuniary externalities, and the design of optimal financial policy are unknown to date.

In this paper, we propose a model of Sudden Stops with liability dollarization (SSLD). Intermediaries raise funds abroad in units of tradable goods but lend them out to domestic agents in units of the aggregate consumption good, represented by a CES composite of tradables and nontradables. As in standard SS models, the value of newly issued debt cannot exceed a fraction of the market value of income in units of tradables, including income from the tradables and nontradables sectors. In order to focus on the effects of liability dollarization on domestic debt, we assume that there are no other frictions in financial intermediation. Banks simply arbitrage the cost of raising funds abroad v. the expected return of domestic loans. Bank liability is unlimited and there are no restrictions on equity issuance or dividends, so that bank failures do not play a role in crisis dynamics in the model. We show that the Fisherian amplification mechanism and the characteristics of optimal financial policies change significantly relative to standard SS models, providing both analytical results and quantitative findings.

Since the SSLD model features the same collateral constraint as the standard SS models, it includes the same pecuniary externality operating via the response of future collateral values to current debt decisions. In addition, by introducing liability dollarization we add a second pecuniary externality that results from the fact that neither financial intermediaries nor domestic borrowers internalize the effects of their decisions on the prices of aggregate consumption and domestic debt (i.e. the real exchange rate and the domestic real interest rate respectively). To distinguish the two externalities, we refer to this new externality as the "intermediation externality" and to the one from the standard SS models as the "macroprudential externality."

Under perfect foresight, we show that the no-arbitrage condition of financial intermediaries implies that liability dollarization matters for the decentralized equilibrium only inasmuch as it
changes the burden of repayment of the outstanding domestic debt that the economy starts with at date 0 . The non-financial wealth of the tradables sector (i.e. the present value of tradables income) is unchanged, but the financial wealth (the interest and principal of the date-0 debt repayment in units of tradables) can be higher or lower in the SSLD model than in the SS model. For a given initial debt in the SS model, there is a threshold value of the initial debt in the SSLD model that supports the same equilibrium in both models, because at this threshold the debt burden in units of tradables is the same, and thus total wealth and hence allocations and relative prices are the same. If the SSLD model debt is lower (higher) than this threshold, tradables consumption in the SSLD model rises (falls) above that in the SS model, but by less than the amount of the reduction in debt. This is because the lower (higher) debt increases (reduces) demand for tradables, which rises (lowers) the relative prices of nontradables and aggregate consumption (i.e. the real exchange rate), thereby increasing (reducing) the burden of repaying the initial debt in the SSLD model. ${ }^{5}$

If the collateral constraint binds under perfect foresight, Sudden Stops are always less severe in the SSLD than in the SS model, because as the relative prices of nontradables and aggregate consumption fall, the burden of the outstanding debt repayment falls, and the resulting additional resources allow the SSLD model to support a higher level of constrained consumption of tradables. Intuitively, liability dollarization introduces an endogenous hedging mechanism that lowers the burden of debt repayment when Sudden Stops hit. In addition, multiplicity of equilibria with a binding collateral constraint is less likely to occur, because the condition required for its existence is more difficult to satisfy.

Liability dollarization has more significant implications in a stochastic environment. Three important effects are at work. First, non-state-contingent credit contracts with repayment at date $t$ are signed and priced at date $t-1$ based on an expected consumption price (i.e. at an ext-ante real interest rate based on an expected real exchange rate) for date $t$. Ex-post deviations of the actual date- $t$ real exchange rate (or equivalently in the ex-post real interest rate) from this expectation induce non-insurable variations in the burden of debt repayment, with similar characteristics as in the perfect-foresight case: When the realized real exchange rate is stronger (weaker), the date- $t$

[^2]burden in units of tradable goods of repaying debt denominated in units of domestic consumption is higher (lower). This gives additional variability to the net-of-debt-repayment income of agents, which strengthens the precautionary savings motive and thus weakens incentives to borrow. Second, if at date $t$ the real exchange rate is expected to appreciate at $t+1$, the no-arbitrage condition of financial intermediaries implies a higher price for newly issued domestic debt (i.e a lower exante real interest rate), which in turn implies that newly issued debt generates more resources for tradables consumption at date $t .{ }^{6}$ Third, the domestic agents' expected marginal cost of borrowing is lower than in the SS model, because aggregate consumption and its price move together, inducing a negative conditional co-variance between the marginal utility of consumption and the ex-post real interest rate in units of domestic consumption at $t+1$. This effect operates as an incentive for risk-taking, which strengthens incentives to borrow. Moreover, this risk-taking incentive is stronger when the expected real exchange rate is lower. In summary, these three effects reflect the fact that under liability dollarization, the borrowers' decisions are influenced by fluctuations in ex-ante and ex-post real interest rates.

In the period in which a Sudden Stop hits the stochastic SSLD model, the impact effects are driven by the same debt-burden-reduction mechanism as in the deterministic case. In fact, for common values of endowment income and outstanding debt at date $t$ when the credit constraint binds, the stochastic and deterministic variants of the SSLD model produce identical allocations. Then, since under perfect foresight Sudden Stops in the SSLD model are always milder than in the SS model, it would seem reasonable to expect milder Sudden Stops in stochastic SSLD models as well. Comparing their equilibrium paths in the stochastic stationary state, however, the frequency and severity of Sudden Stops can be greater in either the SSLD or SS models, because of the different incentives to borrow at work, which in turn imply that the two setups generally arrive at Sudden Stop states with different debt and leverage levels and triggered by shocks of different magnitudes, and thus with different long-run Sudden Stop probabilities. For example, if the risktaking and debt-price effects are sufficiently strong, the SSLD economy visits states with higher debt more frequently than the SS economy.

[^3]Alternatively, consider a situation in which at date $t$ there is some probability of a Sudden Stop at $t+1$. In this case, the expectation as of date $t$ of a collapse in the real exchange rate at $t+1$ if the Sudden Stop occurs reduces the price of domestic debt at $t$, which (ceteris paribus) weakens incentives to borrow in anticipation of the price drop. In addition, the expected marginal cost of borrowing rises, because the conditional covariance between marginal utility and the ex-post real interest rate at $t+1$ rises as the expected real exchange rate falls (i.e. the risk-taking incentive weakens). These effects would work as an endogenous offsetting mechanism making Sudden Stop effects weaker. On the other hand, if there is a Sudden Stop at $t$ and agents expect some recovery of the real exchange rate for $t+1$, the opposite occurs and the incentives to borrow strengthen, increasing the shadow value of the binding collateral constraint at date $t$.

In the normative analysis, we show that the optimal macroprudential policy under commitment becomes time-inconsistent in the SSLD model, in sharp contrast with standard SS models in which it is time-consistent. This is because in the standard model future consumption allocations are irrelevant for the price of nontradables, and hence borrowing capacity, at present, whereas in the SSLD model future consumption matters for current prices because of endogenous fluctuations in interest rates. The optimal policy tightens access to debt when expectations of real appreciation increase, and can be decentralized either as a capital control (i.e. a tax explicitly on inflows of capital) or a tax on domestic debt. As in the standard SS model, the distinction between foreign and domestic lenders is immaterial, and hence while the externalities provide a justification for financial regulation, they do not justify the use of capital controls per-se as a policy discriminating domestic v . foreign lenders.

Since time-inconsistency implies that the optimal policy under commitment lacks credibility, we study a conditionally-efficient optimal policy problem that is time-consistent by construction. In this case, the regulator is required to maintain the same feasible set of borrowing positions as in the absence of regulation, which requires it to support the same equilibrium pricing function for domestic debt. In this case, decentralizing the optimal policy does require two instruments: One to regulate domestic credit (e.g. a tax on domestic debt) and one to make policy credible by maintaining conditional efficiency (e.g. capital controls on inflows to financial intermediaries). Hence, in this case the SSLD model differs from the SS model because it provides a justification for
taxing capital inflows at a different, well-defined rate from that levied on domestic credit, although the latter may be taxed at a positive or negative rate.

We explore the model's quantitative implications using a baseline calibration to Argentina in line with calibrations of SS models already provided in the literature (e.g. Bianchi (2011), Bianchi et al. (2016)). The quantitative analysis yields four key results: (i) The risk-taking and debt-price effects under liability dollarization are strong and result in larger debt holdings, but the implicit hedge reducing the burden of debt repayment reduces the severity of Sudden Stops; (ii) relative to the empirical regularities of Sudden Stops, the SSLD model yields reversals in consumption, the real exchange rate and the current account closer to their data counterparts; (iii) optimal, time-consistent policies make Sudden Stops a zero-probability event, yield a welfare gain of $0.5 \%$ on average, but require complex, non-linear policy rules that imply a median tax on domestic debt of $5.8 \%$ but also a median subsidy on capital inflows of around $12 \%$; and (iv) simple policy rules optimized to maximized welfare yield lower debt taxes of $2 \%$ for constant taxes and $3.6 \%$ for the average of a Taylor-rule-like rule using debt taxes to target debt, both with a welfare-maximizing tax on capital inflows of $0.5 \%$, but they are also much less effective at reducing the frequency and severity of crises and yield smaller welfare gains than the optimal policy. Additionally, setting simple taxes and capital controls to ad-hoc values can reduce welfare significantly.

The rest of the paper proceeds as follows: Section 2 describes the model and characterizes the decentralized equilibrium in the absence of policy intervention. Section 3 studies optimal financial policy of a social planner acting under commitment and for a conditionally-efficient regulator that takes the debt pricing function as given. Section 4 conducts the quantitative analysis. Section 5 provides conclusions.

## 2 A Model of Suddent Stops with Liability Dollarization

We propose a model of Sudden Stops in which banks raise funds in world capital markets at the standard world-determined real interest rate in units of tradable goods, but lend them out in the domestic economy with non-state-contingent debt instruments denominated in units of domestic
consumption, defined by a CES aggregate, and requiring borrowers to meet a debt-to-incomeratio constraint. Intermediaries are risk neutral and do not face any other financial frictions, and therefore they are willing to lend as long as the price of domestic debt implies an expected return that matches the world real interest rate. In turn, the expected return at $t+1$ on domestic debt sold at $t$ depends on the expected relative price of the CES composite good at $t+1$ (which is also the expected real exchange rate, since we assume purchasing power parity in tradable goods). Goods markets are competitive and the prices of traded goods and the world real interest rate are taken as given from world markets.

### 2.1 Private agents

Consider a small open economy where a representative agent consumes tradable goods ( $c^{T}$ ) and nontradable goods $\left(c^{N}\right)$. Preferences are given by a standard expected utility function with period utility defined as a constant-relative-risk-aversion (CRRA) function of a composite good $c_{t}$ :

$$
\begin{equation*}
\mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right), \quad u\left(c_{t}\right)=\frac{c_{t}^{1-\gamma}}{1-\gamma} . \tag{1}
\end{equation*}
$$

$\mathbb{E}(\cdot)$ is the expectation operator, $\beta$ is the discount factor, and $\gamma$ is the coefficient of relative risk aversion. The composite good is a CES agregator:

$$
\begin{equation*}
c_{t}=\left[\omega\left(c_{t}^{T}\right)^{-\eta}+(1-\omega)\left(c_{t}^{N}\right)^{-\eta}\right]^{-\frac{1}{\eta}}, \eta>-1, \omega \in(0,1) . \tag{2}
\end{equation*}
$$

The elasticity of substitution between $c_{t}^{T}$ and $c_{t}^{N}$ is given by $1 /(1+\eta)$.

The agent receives stochastic endowments of tradable and nontradable goods $y_{t}^{T}$ and $y_{t}^{N}$, and can trade non-state-contingent bonds $b_{t}^{c}$ denominated in units of $c_{t}$ at a price $q_{t}^{c}$ with financial intermediaries. The relative price of nontradable goods in units of tradables is denoted $p_{t}^{N}$, and the relative price of the composite good $c_{t}$ in units of tradables is denoted $p_{t}^{c}$. Following standard practice in dependent economy models (see Obstfeld and Rogoff (1996) p. 227), we apply the Duality Theory of consumer choice to characterize this price as the price index that corresponds to the minimum expenditure $c_{t}^{T}+p_{t}^{N} c_{t}^{N}$ such that $c_{t}=1$. Given the CES structure of the composite
good, the price index is given by:

$$
\begin{equation*}
p_{t}^{c}=\left[\omega^{\frac{1}{1+\eta}}+(1-\omega)^{\frac{1}{1+\eta}}\left(p_{t}^{N}\right)^{\frac{\eta}{1+\eta}}\right]^{\frac{1+\eta}{\eta}}, \eta>-1, \omega \in(0,1) \tag{3}
\end{equation*}
$$

This relative price is the economy's consumer-price-based measure of the real exchange rate, because foreign prices are normalized to 1 for simplicity and purchasing power parity in tradables holds, and hence the ratio of domestic to foreign consumer prices is the same as $p_{t}^{c}$. Notice also a property of this relative price that will be important for the analysis that follows: $p_{t}^{c}$ is a monotonic, increasing function of $p_{t}^{N}$.

Choosing the price of tradables as the numeraire, the agent's budget constraint is:

$$
\begin{equation*}
q_{t}^{c} p_{t}^{c} b_{t+1}^{c}+c_{t}^{T}+p_{t}^{N} c_{t}^{N}=p_{t}^{c} b_{t}^{c}+y_{t}^{T}+p_{t}^{N} y_{t}^{N} \tag{4}
\end{equation*}
$$

The left-hand-side of this expression shows the uses of the agent's income in units of tradables: purchases (sales) of bonds that require (generate) resources by the amount $p_{t}^{c} q_{t}^{c} b_{t+1}^{c}$ when $b_{t+1}^{c}>0$ $\left(b_{t+1}^{c}<0\right)$, plus total expenditures in consumption of tradables and nontradables. The right-hand-side shows the sources of the agent's income: Income from maturing bond holdings $p_{t}^{c} b_{t}^{c}$ (or repayment of debt if $b_{t}^{c}<0$ ), the realization of the endowment of tradables $y_{t}^{T}$, and the value of the realization of the nontradables endowment in units of tradables $p_{t}^{N} y_{t}^{N}$. The stochastic processes of the endowments follow standard Markov processes to be specified later.

Borrowing requires collateral and only a fraction of the agent's income is pledgeable as collateral. As a result, the representative agent cannot borrow more than a fraction $\kappa$ of total income in units of tradables:

$$
\begin{equation*}
q_{t}^{c} p_{t}^{c} b_{t+1}^{c} \geq-\kappa\left(y_{t}^{T}+p_{t}^{N} y_{t}^{N}\right) \tag{5}
\end{equation*}
$$

This constraint can be interpreted as the result of enforcement or institutional frictions by which lenders are only able to harness a fraction $\kappa$ of a defaulting borrower's income, or borrowers can only pledge a fraction $\kappa$ of their income as collateral. It can also be viewed as resulting from conventional practices in credit markets, such as the loan-to-income ratios used to limit household credit directly or indirectly via credit scores that penalize high debt-income ratios.

The representative agent chooses the stochastic sequences $\left\{c_{t}^{T}, c_{t}^{N}, b_{t+1}^{c}\right\}_{t \geq 0}$ to maximize (1) subject to (4) and (5), taking $b_{0}$ and $\left\{p_{t}^{N}, p_{t}^{c}, q_{t}^{c}, y_{t}^{T}, y_{t}^{N}\right\}_{t \geq 0}$ as given.

### 2.2 Financial Intermediation

We assume that there are deep-pockets, risk-neutral financial intermediaries who float bonds in international markets at a world-determined price $q_{t}^{*}$ (i.e. the inverse of the gross world real interest rate) to fund purchases of the bonds that provide domestic financing at the price $q_{t}^{c}$. These intermediaries arbitrage the return on domestic lending v . their funding cost, which implies that domestic debt is priced according to the following no-arbitrage condition: ${ }^{7}$

$$
\begin{equation*}
q_{t}^{c}=\frac{q_{t}^{*} \mathbb{E}_{t}\left[p_{t+1}^{c}\right]}{p_{t}^{c}} \tag{6}
\end{equation*}
$$

Hence, the price at which domestic agents can sell new debt at date $t$ is determined by the ratio of the conditional expectation of consumption prices at $t+1$ to observed prices at $t$ (i.e. the expected rate of real appreciation). This price of debt has associated with it the ex-ante domestic real interest rate $R_{t+1}^{c} \equiv 1 / q_{t}^{c}=\frac{R_{t+1}^{*} p_{t}^{c}}{\mathbb{E}_{t}\left[p_{t+1}^{c}\right]}$, where $R_{t+1}^{*}=1 / q_{t}^{*}$ is the world real interest rate. Similarly, the ex-post (i.e. after $p_{t+1}^{c}$ is observed) debt price and real interest rate are defined as $\tilde{q}_{t}^{c} \equiv \frac{q_{t}^{c} p_{t}^{c}}{p_{t+1}^{c}}$ and $\tilde{R}_{t+1}^{c} \equiv 1 / \tilde{q}_{t}^{c}=\frac{R_{t+1}^{c} p_{t+1}^{c}}{p_{t}^{c}}$, which are both contingent on the realization of $p_{t+1}^{c}$. The difference between these ex-ante and ex-post interest rates will play a central role later for characterizing the effects of liability dollarization.

Except for liability dollarization, this is a frictionless characterization of financial intermediation. It facilitates both the theoretical analysis and the numerical solution of the model significantly, because it yields a pricing condition for domestic debt that depends only on expected and current prices, while it still introducing the important effects of the intermediation externality that are absent from the standard SS model, in which both sides of the intermediaries' balance sheet are in units of tradables. On the other hand, studying a setup in which financial intermediaries face additional relevant frictions would be worth pursuing. In particular, we assume here that interme-

[^4]diaries have unrestricted access to world financial markets and that intermediation does not incur any costs other than the funding cost $R^{*}$. Intermediaries face no origination costs in transforming hard-currency borrowing into domestic-currency lending, they can pay negative dividends (issue new equity) and can always cover a shortfall between income from loans paid by domestic agents and repayment to foreign creditors (i.e. negative equity) with additional external borrowing. ${ }^{8}$ Moreover, since intermediaries face an exogenous cost of funding, they are effectively indifferent between funding loans at the margin with equity or foreign capital inflows.

### 2.3 Competitive Equilibrium \& Comparison with Standard SS Models

In the absence of policy intervention, a competitive equilibrium for the SSLD model is given by sequences of allocations $\left\{c_{t}^{T}, c_{t}^{N}, b_{t+1}^{c}\right\}_{t \geq 0}$, and prices $\left\{p_{t}^{N}, p_{t}^{c}, q_{t}^{c}\right\}_{t \geq 0}$ such that: (a) the representative agent maximizes utility subject to the budget and collateral constraints taking prices as given, (b) the no-arbitrage condition of the financial intermediaries holds, and (c) the marketclearing condition of the market of nontradables $\left(c_{t}^{N}=y_{t}^{N}\right)$ and the resource constraint of tradables $\left(c_{t}^{T}=y_{t}^{T}-q_{t}^{c} p_{t}^{c} b_{t+1}^{c}+p_{t}^{c} b_{t}\right)$ hold.

The equilibrium conditions include the first-order conditions of the agent's problem, the noarbitrage condition of intermediaries, the nontradables market-clearing condition and the tradables resource constraint:

$$
\begin{gather*}
\lambda_{t}=u_{T}(t)  \tag{7}\\
p_{t}^{N}=\left(\frac{1-\omega}{\omega}\right)\left(\frac{c_{t}^{T}}{c_{t}^{N}}\right)^{\eta+1}  \tag{8}\\
\lambda_{t}=\beta \mathbb{E}_{t}\left[\frac{\lambda_{t+1} p_{t+1}^{c}}{q_{t}^{c} p_{t}^{c}}\right]+\mu_{t}  \tag{9}\\
q_{t}^{c} p_{t}^{c} b_{t+1}^{c} \geq-\kappa\left[y_{t}^{T}+p_{t}^{N} y_{t}^{N}\right], \quad \text { with equality if } \mu_{t}>0  \tag{10}\\
q_{t}^{c} p_{t}^{c}=q_{t}^{*} \mathbb{E}_{t}\left[p_{t+1}^{c}\right]  \tag{11}\\
c_{t}^{N}=y_{t}^{N} \tag{12}
\end{gather*}
$$

[^5]\[

$$
\begin{equation*}
c_{t}^{T}=y_{t}^{T}-q_{t}^{c} p_{t}^{c} b_{t+1}^{c}+p_{t}^{c} b_{t} \tag{13}
\end{equation*}
$$

\]

where $\lambda_{t}$ and $\mu_{t}$ are the non-negative Lagrange multipliers on the budget and credit constraints respectively, and $u_{T}(t) \equiv u^{\prime}\left(c_{t}\right) \partial c_{t} / \partial c_{t}^{T}$ is the marginal utility of consumption of tradables. Note that conditions (8) and (12), and the consumption price index (3), can be used to express the price of nontradables and the aggregate price index as functions of $c_{t}^{T}$ and $y_{t}^{N}$, denoted $p^{N}\left(c_{t}^{T}, y_{t}^{N}\right)$ and $p^{c}\left(c_{t}^{T}, y_{t}^{N}\right)$ respectively. Moreover, the CES structure of preferences implies that these functions are monotonic and increasing in $c_{t}^{T}$, hence $p^{N^{\prime}}(t) \equiv \frac{\partial p^{N}(t)}{\partial c_{t}^{T}}>0$ and $p^{c \prime}(t) \equiv \frac{\partial p^{c}(t)}{\partial c_{t}^{T}}>0$.

## (a) Comparing deterministic equilibria

The above equilibrium conditions can be used to study how equilibrium prices and allocations and the Fisherian debt-deflation mechanism that drives Sudden Stops differ between the SSLD model and the standard SS model. Consider first the two models under perfect foresight. In this case, ex-ante and ex-post real interest rates and debt prices are the same, and algebraic manipulation of conditions (7), (9) and (11)-(13) reduces to the following Euler equation and intertemporal resource constraint (assuming a constant world real interest rate to keep notation simple):

$$
\begin{gather*}
u_{T}(t)=\beta R^{*}\left[u_{T}(t+1)\right]+\mu_{t}  \tag{14}\\
\sum_{t=0}^{\infty} R^{*-t} c_{t}^{T}=\sum_{t=0}^{\infty} R^{*-t} y_{t}^{T}+p_{0}^{c} b_{0}^{c} \tag{15}
\end{gather*}
$$

These two conditions, together with (8), (10) and (12) characterize fully the SSLD equilibrium under perfect foresight. Following Mendoza (2005), we simplify the analysis by assuming that $\beta R^{*}=1$, initial bond holdings are negative (i.e. the economy starts with some debt), and initial tradables income is lower than in the future so that $b_{1}^{c}<0$, and we study wealth-neutral shocks that reduce the tradables endowment in the first period so as to induce agents to borrow more. For a sufficiently large shock, the collateral constraint binds, but for smaller shocks it does not.

If the collateral constraint does not bind, it is straightforward to verify that the conditions that characterize the perfect-foresight equilibria of the SSLD and SS models are almost identical, except for one difference: In the expression that defines wealth in the right-hand-side of (15), financial wealth is given by the term $p_{0}^{c} b_{0}^{c}$ in the SSLD model, v. $b_{0}$ in the SS model (with bonds denominated
in tradables units). These two can differ because, for given values of the exogenous initial conditions $b_{0}^{c}$ and $b_{0}$, the equilibrium value of the initial price $p_{0}^{c}$ determines whether the burden of repayment of the initial debt is higher in the SSLD or the SS case.

Taking the equilibrium price of nontradables $p^{N, S S}$ from a solution of the SS model for a given $b_{0}$, we can compute the corresponding value of the consumption price index $p^{c, S S}$ and then define a threshold initial debt level in the SSLD model $\tilde{b}_{0}^{c} \equiv p^{c, S S} b_{0}$ such that the two models produce the same perfect-foresight equilibrium. If $b_{0}^{c}<\tilde{b}_{0}^{c}\left(b_{0}^{c}>\tilde{b}_{0}^{c}\right)$, the SS model yields higher (lower) tradables consumption and prices than the SSLD model. ${ }^{9}$ Consumption rises (falls) by less than the reduction (increase) in debt because the increase (fall) in $p^{c}$ increases (reduces) the debt repayment burden and hence offsets some of the effect of the lower (higher) debt. Liability dollarization does not have any other effects, and in particular it does not alter the tradeoff between marginal costs and benefits of borrowing reflected in the Euler equation (14).

If the reduction in $y_{0}^{T}$ is sufficiently large to make the collateral constraint bind at $t=0$, a Sudden Stop occurs. Condition (13) implies that $c_{t}^{T}$ falls, because access to debt to sustain tradables consumption is constrained. Then it follows from condition (8) that $p_{t}^{N}$ falls to clear the nontradables market. This generates a further tightening of the collateral constraint, because it reduces the value of collateral provided by the nontradables endowment in condition (10). Formally, the date- 0 allocations and prices are determined by a nonlinear equation in $c_{0}^{T}$ that results from imposing condition (10) with equality in the resource constraint (13):

$$
\begin{equation*}
c_{0}^{T}=y_{0}^{T}+\kappa\left[y_{0}^{T}+p_{0}^{N}\left(c_{0}^{T}\right) y_{0}^{N}\right]+p_{0}^{c}\left(c_{0}^{T}\right) b_{0}^{c} \tag{16}
\end{equation*}
$$

This condition is again almost the same that determines consumption in a Sudden Stop in the SS model, except for the debt repayment term $p_{0}^{c}\left(c_{0}^{T}\right) b_{0}^{c}$, which in the SS model is just $b_{0}$. By the same argument as before, the Sudden Stop equilibrium prices of the SS model can be used to set the exogenous value of $b_{0}^{c}$ so as to make the SS and SSLD solutions the same. But assume instead that the initial condition was the value $\tilde{b}_{0}^{c}$ that sustained identical stationary equilibria in the SS and

[^6]SSLD model when the income shock was not large enough to trigger the constraint. In this case, the Sudden Stop equilibria of the two economies differ.

Figure 1 illustrates the determination of the two equilibria in the $\left(c^{T}, p^{N}\right)$ space, in a manner analogous to Figure 2 in Mendoza (2005). The $P P$ curve is the marginal rate of substitution in consumption of tradables and nontradables, which given the constant endowment of nontradables yields a convex function that maps $c^{T}$ into $p^{N}$ (this curve is the same for the SS and SSLD models). The various BB curves show the value of $p^{N}$ that corresponds to a value of $c^{T}$ such that the collateral constraint holds with equality and the tradables resource constraint is satisfied, for the SS and SSLD models and for each under different values of $y_{0}^{T}$. In each case, equilibrium is reached where the $P P$ curve and the relevant BB curve intersect.

Figure 1: Sudden Stops under Perfect Foresight


Mendoza showed that in the SS model, the $\mathrm{BB}^{S S}$ curves are linear functions of $c_{0}^{T}$ with an horizontal intercept given by $I^{S S} \equiv(1+\kappa) y_{0}^{T}+b_{0}$ and a slope of $m^{S S} \equiv 1 /\left(\kappa y^{N}\right)$. For the SSLD model, we show in the Appendix that the $\mathrm{BB}^{S S L D}$ curves are convex functions of $c_{0}^{T}$ with an horizontal intercept given by $I^{S S L D} \equiv(1+\kappa) y_{0}^{T}+\omega^{1 / \eta} \tilde{b}_{0}^{c}$ and a slope of $m^{S S L D} \equiv\left[1-p^{c \prime}(t) \tilde{b}_{0}^{c}\right] /\left(\kappa y^{N}\right)$. Notice that again the difference between the SS and SSLD models is due to differences in the repayment burden of the initial debt induced by changes in the relative price of consumption. The
term $\omega^{1 / \eta}$ in the intercept of the SSLD model is the lower bound of $p^{c}$ that is reached when $p^{N}=0$. Since at $\tilde{b}_{0}^{c}$ we have the same debt in units of tradables in the SS and SSLD models at the higher prices supported in the stationary equilibrium with the constraint not being binding, the intercept of the SSLD model must be to the right of the one in the SS model (since the same debt stock is valued at the minimum price in the intercept of the SSLD model).

The $\mathrm{BB}^{S S, 0}, \mathrm{BB}^{S S L D, 0}$ curves are for the threshold value of $y_{0}^{T}$ such that the constraint allows for just enough debt to support the stationary equilibrium without Sudden Stop. Hence, by construction (again given $\tilde{b}_{0}^{c}$ ) these curves cut the $P P$ curve at the same point A and yield the same equilibrium values of $c^{T}$ and $p^{N}$ in SS and SSLD. The $\mathrm{BB}^{S S, 1}, \mathrm{BB}^{S S L D, 1}$ curves are for a lower $y_{0}^{T}$, which shifts the BB curves of both models to the left, triggering the credit constraint and causing a Sudden Stop.

The main point of Figure 1 is to show that Sudden Stops under perfect foresight are milder in the SSLD model. The Sudden Stop equilibria are reached at points B and C for the SSLD and SS model respectively. Since the $\mathrm{BB}^{S S L D, 1}$ curve is always steeper than the $\mathrm{BB}^{S S, 1}$ curve and has a higher horizontal intercept, and since for given $y_{0}^{T}$ the two curves always intersect at point A, $\mathrm{BB}^{S S L D, 1}$ must cut the PP curve to the right of where $\mathrm{BB}^{S S, 1}$ cuts it. ${ }^{10}$ This implies that in the Sudden Stop of the SSLD economy, tradables consumption and relative prices are higher than in the SS economy. Both equilibria are Sudden Stops, because financial amplification via the deflation of the value of collateral causes a sudden drop from the stationary consumption and prices at point A, but the drops in the SSLD model are always milder. The intuition is simple: In the SSLD economy, the fall in the real exchange rate associated with a Sudden Stop reduces $p_{0}^{c}$, and hence the burden of repaying $\tilde{b}_{0}^{c}$ in terms of tradable goods falls, providing additional resources for consumption of tradables.

It is also worth noting that multiplicity of Sudden Stop equilibria remains possible under liability dollarization, but is less likely. Mendoza (2005) and Schmitt-Grohé and Uribe (2017) study multiplicity in the SS model. In terms of Figure 1, multiplicity in Sudden Stop equilibria emerges if the parameters of the PP and BB functions are such that the two curves intersect twice in the

[^7]region to the left of point A. This requires relatively high values of $\kappa$ and/or relatively low elasticities of substitution between tradables and nontradables and low ratios of tradables-to-nontradables consumption. In the SSLD model, however, the BB curve becomes convex, and this makes it less likely that SS and PP can intersect in the relevant range. The formal condition for multiplicity in the SSLD model is derived and compared with that of the SS model in the Appendix.

Summing up, under perfect foresight, liability dollarization introduces one relatively benign implication. It alters equilibrium allocations and prices only through a valuation effect that changes the burden of repaying the initial debt depending on the equilibrium value of the initial real exchange rate. Because of this valuation effect, Sudden Stops yield smaller declines in consumption and prices than in standard SS models.

## (b) Comparing stochastic equilibria

Liability dollarization has more significant implications under uncertainty. In particular, since debt is non-state-contingent, liability dollarization introduces three key effects that result from the fact that borrowers are affected by fluctuations in ex-ante and ex-post domestic real interest rates, while intermediaries are only affected by ex-ante real interest rates.

1. Fluctuations in the burden of outstanding debt repayment: At any date $t$, the burden of repaying $b_{t}^{c}$, which was contracted at $t-1$, changes with the realized equilibrium value of $p_{t}^{c}$. Qualitatively, this effect is akin to the one present under perfect foresight, but in the stochastic model it causes ex-post, non-insurable fluctuations in income disposable for tradables consumption whenever the actual date- $t$ price deviates from the expected value of the price at $t-1$ (which was used to set the price $q_{t-1}^{c}$ and to choose $b_{t}^{c}$ ). If the realized price is higher (lower), the ex-post domestic real interest rate $\tilde{R}_{t}^{c}$ rises (falls) and the burden of debt repayment rises (falls). Moreover, these fluctuations increase income volatility, which strengthens incentives for precautionary savings (weakens incentives to accumulate debt).
2. Fluctuations in debt prices and resources generated by newly issued debt: Using the noarbitrage condition of intermediaries, the amount of tradable goods resources generated by debt can be written as $-q_{t}^{c} p_{t}^{c} b_{t+1}^{c}=-q_{t}^{*} \mathbb{E}_{t}\left[p_{t+1}^{c}\right] b_{t+1}^{c}$. This implies that a given amount
of newly issued debt $b_{t+1}^{c}<0$ generates more resources for tradables consumption when consumption prices are expected to be higher (i.e. when the real exchange rate is expected to appreciate). Hence, an expected real appreciation (depreciation) causes a decline (increase) in the ex-ante real interest rate $R_{t+1}^{c}$ (or equivalently an increase (fall) in the price of new bond issuances by domestic agents) that incentivizes increased (reduced) borrowing.
3. Risk-taking incentive The marginal cost of borrowing faced by domestic agents falls because of the positive co-movement between consumption and prices. To derive this result, notice that the Euler equation (9) can be simplified as follows:

$$
\begin{equation*}
u_{T}(t)=\beta \mathbb{E}_{t}\left[u_{T}(t) \tilde{R}_{t+1}^{c}\right]+\mu_{t} \tag{17}
\end{equation*}
$$

Using the lenders' arbitrage condition (6), this expression can be re-written as:

$$
\begin{equation*}
u_{T}(t)=\beta R_{t+1}^{*} \mathbb{E}_{t}\left[u_{T}(t+1)\right]+\beta \operatorname{Cov}_{t}\left(u_{T}(t+1), \tilde{R}_{t+1}^{c}\right)+\mu_{t} \tag{18}
\end{equation*}
$$

The marginal cost of borrowing in the right-hand-side of this expression includes the risk term $\beta \operatorname{Cov}_{t}\left(u_{T}(t+1), \tilde{R}_{t+1}^{c}\right)$, with a sign that depends on the sign of the conditional covariance between the marginal utility of tradables and the ex-post domestic real interest rate. Using the definition of $\tilde{R}_{t+1}^{c}$, the lenders' arbitrage condition and the properties of the covariance operator, we can re-write the covariance as $\operatorname{Cov}_{t}\left(u_{T}(t+1), \tilde{R}_{t+1}^{c}\right)=\frac{1}{q_{t}^{*}\left[\mathbb{E}_{t}\left[p_{t+1}^{c}\right]\right.} \operatorname{Cov}_{t}\left(u_{T}(t+\right.$ 1), $p_{t+1}^{c}$ ). Since $p^{c}$ is a monotonic, increasing function of $c^{T}$, a lower $c_{t+1}^{T}$ increases the marginal utility of tradables and decreases $p_{t+1}^{c}$, so the covariance is negative. ${ }^{11}$ In fact, since $c_{t+1}^{T}$ and $p_{t+1}^{c}$ are perfectly correlated, the covariance term reduces to: $\operatorname{Cov}_{t}\left(u_{T}(t+1), \tilde{R}_{t+1}^{c}\right)=$ $\frac{-\sigma_{t}\left(u_{T}(t+1)\right) \sigma_{t}\left(p_{t+1}^{c}\right)}{\left.q_{t}^{\mathbb{E}_{t}} \dagger p_{t+1}^{c}\right\rfloor}$, where $\sigma_{t}\left(u_{T}(t+1)\right)$ and $\sigma_{t}\left(p_{t+1}^{c}\right)$ are conditional standard deviations. Hence, the risk-taking borrowing incentive strengthens when tradables marginal utility is more variable and/or when the coefficient of variation of the real exchange rate $\left(\frac{\sigma_{t}\left(p_{t+1}^{c}\right)}{\mathbb{E}_{t}\left(p_{t+1}^{c}\right)}\right)$ rises. ${ }^{12}$ This result also implies that, for given standard deviations of marginal utility and prices, the

[^8]risk-taking incentive weakens when the real exchange rate is expected to appreciate. Hence, an expected real appreciation triggers opposing effects on borrowing incentives: It weakens the risk-taking incentive but strengthens the debt-price incentive (i.e. newly issued bonds have a higher price as the ex-ante real interest rate falls).

The above effects are present solely because of liability dollarization, so they are present even without the collateral constraint. Since the three effects move incentives to borrow in different directions, whether the SS or the SSLD model yield larger debt positions, smoother consumption, larger or more frequent Sudden Stops, etc. depends on their relative strength, and this is a question that is be answered with quantitative analysis, as we do in Section 4.

Although Sudden Stops will differ in magnitude and frequency across the SSLD and SS models, it is important to note that in periods in which the constraint binds, allocations and prices still differ only because of the valuation effect altering the repayment burden of the outstanding debt. This is because if the constraint binds at date $t, c_{t}^{T}$ is determined by the same non-linear equation (16) as in the deterministic case, except that it now holds for any date $t$ and any ( $b_{t}^{c}, y_{t}^{T}$ ) pair in which the collateral constraint binds. Still, the magnitude and frequency of Sudden Stops will differ depending on the outstanding debt $\left(b_{t}^{c}\right)$ and the size of income shocks $\left(y_{t}^{T}, y_{t}^{N}\right)$ needed to trigger the constraint in each economy. The outstanding debt that triggers Sudden Stops is endogenous and depends on the history of previous income shocks and optimal debt decisions, which differ in the two models because of the three effects listed above. Similarly, the frequency of Sudden Stops will differ depending on the long-run probability with which each economy reaches states with enough debt to trigger a Sudden Stop. If, for example, the risk-taking and debt-price effects dominate, the SSLD economy will accumulate more debt and could be more likely to experience Sudden Stops than the SS economy, even though when those Sudden Stops occur the decline in prices provides some relief by lowering the repayment burden. In addition, credit in the SSLD economy may grow faster in the run-up to Sudden Stops if agents expect higher real exchange rates, which reduce ex-ante real interest rates, and/or if the risk-taking incentive strengthens. Conversely, if stronger precautionary savings incentives dominate and/or agents expect lower real exchange rates in the pre-crisis periods of the SSLD economy, debt will grow more slowly and may in general be lower than in the SS economy, leading to less frequent and less severe Sudden Stops. Post-Sudden-Stop
dynamics will also differ, because the SSLD and SS economies will transit out of states in which the collateral constraint binds with different levels of debt, and hence different allocations and prices.

### 2.4 Capital Controls and Domestic Debt Taxes

In the normative analysis of the next Section, we use two policy instruments to decentralize socially optimal allocations: Capital controls (i.e. taxes on the intermediaries' inflows of foreign capital) and domestic debt taxes (taxes on domestic borrowing). Capital controls are modeled as a tax $\theta_{t}$ that raises the interest rate at which intermediaries borrow from abroad above $R_{t}^{*}$ (i.e. it lowers the price of bonds sold abroad below $q_{t}^{*}$ ). With this tax in place, the intermediaries' no-arbitrage condition becomes:

$$
\begin{equation*}
q_{t}^{c}=\frac{q_{t}^{*}}{\left(1+\theta_{t}\right)} \frac{\mathbb{E}_{t}\left[p_{t+1}^{c}\right]}{p_{t}^{c}} \tag{19}
\end{equation*}
$$

The revenue generated by this tax is rebated to intermediaries as a lump-sum transfer, which can also be a lump-sum tax if $\theta_{t}<0$. Notice that the tax is known at the moment of issuing bonds, and is paid with the bond repayment.

The tax on domestic debt $\tau_{t}$ increases the cost of borrowing for domestic agents. If this tax is used, the budget and collateral constraints of the representative agent become:

$$
\begin{align*}
q_{t}^{c} p_{t}^{c} b_{t+1}^{c}+c_{t}^{T}+p_{t}^{N} c_{t}^{N} & =p_{t}^{c} b_{t}^{c}\left(1+\tau_{t}\right)+y_{t}^{T}+p_{t}^{N} y_{t}^{N}+T_{t}  \tag{20}\\
q_{t}^{c} p_{t}^{c} b_{t+1}^{c} & \geq-\kappa\left(y_{t}^{T}+p_{t}^{N} y_{t}^{N}\right) \tag{21}
\end{align*}
$$

where $T_{t}$ is a lump-sum rebate of the revenue generated by this tax (or a lump-sum tax if $\tau_{t}<0$ ).

If both taxes are used, the intermediaries' no-arbitrage condition implies that the agent's Euler equation for bonds can be expressed as:

$$
\begin{equation*}
u_{T}(t)=\left(1+\tau_{t}\right)\left(1+\theta_{t}\right) \beta \mathbb{E}_{t}\left[u_{T}(t+1) \tilde{R}_{t+1}^{c}\right]+\mu_{t} \tag{22}
\end{equation*}
$$

This condition implies that, unless decentralizing socially optimal allocations requires different taxes on capital inflows and domestic debt, taxing one is equivalent to taxing the other in terms
of their effect on the agent's Euler equation for bonds. What matters is the combined effective tax rate $\left(1+\tau_{t}\right)\left(1+\theta_{t}\right)$, and the particular values of each tax are undetermined. In particular, if the only inefficiency in the unregulated decentralized equilibrium is the standard macroprudential externality due to collateralized domestic debt, the optimal policy can be implemented equally with only domestic debt taxes, only capital controls or a mix of both. This is the case in the standard SS model, because intermediation is inessential and the only inefficiency is the macroprudential externality. As explained earlier, intermediation in the SS setup can be interpreted as frictionless domestic banks that borrow and lend in tradables units, or as the nonfinancial private sector borrowing directly from abroad. Either way, the optimal policy needs to tackle only the inefficiency driving a wedge between the social and private marginal costs of domestic borrowing, and in this case domestic debt taxes and capital controls are equivalent. Hence, the standard SS model of Sudden Stops does not provide a justification for capital controls (i.e. for a policy to discriminate domestic from foreign credit flows).

It is important to note that the above equivalence result holds in part due to the model's stylized formulation of financial intermediation. If issuing domestic loans has a variable cost, for instance, the marginal cost of issuing domestic bonds would be subtracted from the right-hand-side of (19) and this would imply that setting $\theta_{t}$ at a given rate results in a larger increase in effective borrowing costs that setting $\tau_{t}$ at the same rate. Hence, extending the model to introduce realistic frictions in financial intermediation may not only introduce non-neutral balance sheet effects on banks as the real exchange rate moves, but may also provide a justification for capital controls.

## 3 Optimal Financial Policy

We study optimal financial policy following a primal approach by analyzing the allocations attainable to a social planner ( SP ) who chooses the debt of private agents subject to the resource, market-clearing, and collateral constraints, letting goods markets and financial intermediaries operate competitively. ${ }^{13}$ First we study the planner's problem under commitment. As we show below,

[^9]the optimal policy is time-inconsistent in this case, because of the planner's ability to affect the ex-ante domestic interest rate for debt contracted at date $t$ with the consumption of tradable goods planned for date $t+1$. Since time-inconsistency implies that the optimal policy under commitment lacks credibility, we study next a conditionally-efficient social planner's problem that yields time-consistent optimal financial policy. This approach follows Bianchi and Mendoza (2010) in constructing time-consistent policy by requiring the planner to support a pricing function that prevails in the decentralized equilibrium without policy intervention.

### 3.1 Optimal policy under commitment

Since the planner makes borrowing decisions for private agents, they no longer choose $b_{t+1}^{c}$ and do not face the collateral constraint. They choose tradables and nontradables consumption optimally subject to a budget constraint that includes the income from tradables and nontradables, and a lump-sum transfer (tax) from the planner that represents the amount of resources generated by borrowing (repaying). Hence, the first-order conditions of the agents' optimization problem no longer include the Euler equation for bonds (equation (9)) and the borrowing constraint (equation (10)), but the optimality condition equating the marginal rate of substitution in consumption of tradables and nontradables with the relative price of nontradables (equation (11)) still holds. The planner must satisfy this condition because the market of nontradable goods still clears competitively. In addition, the no-arbitrage condition of financial intermediaries must also hold, because these intermediaries are also still operating competitively.

To simplify SP's optimization problem, we use the nontradables market-clearing condition to remove $c_{t}^{N}$ from the problem and the intermediaries' no-arbitrage condition to replace $q_{t}^{c} p_{t}^{c}$ with $q_{t}^{*} \mathbb{E}_{t}\left[p_{t+1}^{c}\right]$ in the budget and borrowing constraints. The social planner's problem under commit-
ment can then be written as:

$$
\begin{align*}
& \max _{\left\{c_{t}^{T}, b_{t+1}^{c}\right\}} \mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\left(c_{t}^{T}, y_{t}^{N}\right)\right)  \tag{23}\\
& \text { s.t. } \\
& q_{t}^{*} \mathbb{E}_{t}\left[p^{c}\left(c_{t+1}^{T}, y_{t+1}^{N}\right)\right] b_{t+1}^{c}+c_{t}^{T}=p^{c}\left(c_{t}^{T}, y_{t}^{N}\right) b_{t}^{c}+y_{t}^{T}  \tag{24}\\
& q_{t}^{*} \mathbb{E}_{t}\left[p^{c}\left(c_{t+1}^{T}, y_{t+1}^{N}\right)\right] b_{t+1}^{c} \geq-\kappa\left(y_{t}^{T}+p^{N}\left(c_{t}^{T}, y_{t}^{N}\right) y_{t}^{N}\right) \tag{25}
\end{align*}
$$

Notice that the planner's resource and borrowing constraints are affected by both the current and expected real exchange rates, or equivalently by the ex-ante and ex-post real interest rates. As we show below, the time-inconsistency of the optimal policy under commitment is due to the fact that future consumption matters for today's ex-ante real interest rate, and thus for today's consumption.

As shown in the Appendix, the first-order conditions of the above problem can be reduced to the following expressions:

$$
\begin{align*}
& \lambda_{t}=\frac{u_{T}(t)+\mu_{t} \kappa p^{N^{\prime}}(t) y_{t}^{N}-p^{c \prime}(t) b_{t}^{c} \frac{q_{t-1}^{*}}{\beta}\left(\lambda_{t-1}-\mu_{t-1}\right)}{1-p^{c \prime}(t) b_{t}^{c}}  \tag{26}\\
& \lambda_{t}=\frac{\beta \mathbb{E}_{t}\left[\lambda_{t+1} p^{c}(t+1)\right]}{q_{t}^{*} \mathbb{E}_{t}\left[p^{c}(t+1)\right]}+\mu_{t}  \tag{27}\\
& q_{t}^{*} \mathbb{E}_{t}\left[p^{c}(t+1)\right] b_{t+1}^{c}+\kappa\left[y_{t}^{T}+p^{N}(t) y_{t}^{N}\right] \geq 0, \quad \text { with equality if } \mu_{t}>0 \tag{28}
\end{align*}
$$

where $p^{c \prime}(t)=\frac{p^{c}(t)(\eta+1)}{\left[1+\left(\frac{\omega}{(1-\omega)\left(p^{N}(t)\right)^{\eta}}\right)^{\frac{1}{\eta+\mathrm{T}}}\right]}$ $c_{t}^{T}$ and $p^{N^{\prime}}(t)=\frac{(\eta+1) p^{N}(t)}{c_{t}^{T}}$.
Lagging condition (27) one period and using it in condition (26) yields:

$$
\begin{equation*}
\lambda_{t}=\frac{u_{T}(t)+\mu_{t} \kappa p^{N^{\prime}}(t) y_{t}^{N}-p^{c \prime}(t) b_{t}^{c}\left(\mathbb{E}_{t-1}\left[\lambda_{t}\right]+\frac{\operatorname{Cov}_{t-1}\left(\lambda_{t}, p^{c}(t)\right)}{\mathbb{E}_{t-1}\left[p^{c}(t)\right]}\right)}{1-p^{c \prime}(t) b_{t}^{c}} \tag{29}
\end{equation*}
$$

This expression is useful for highlighting differences in the social v. private marginal utility of wealth (i.e. the valuation of the marginal benefit of an additional unit of tradables consumption of the planner v. domestic agents in the decentralized equilibrium). Recall that for private agents $\lambda_{t}=u_{T}(t)$, while for the planner the above expression shows important differences reflected in the
additional terms in the numerator and denominator in the right-hand-side of (29).

The second term in the numerator $\left(\mu_{t} \kappa p^{N \prime}(t) y_{t}^{N}\right)$ captures the macroprudential externality operating via the effect of the price of nontradables (i.e. the value of collateral) on borrowing capacity, just as in the standard SS model. In states in which the collateral constraint binds, the social marginal benefit of tradables consumption includes the shadow value of the additional borrowing capacity that higher consumption of tradables provides by increasing the price of nontradables, because the planner internalizes how that price responds to changes in $c_{t}^{T}$. This term is strictly positive because $p_{t}^{N}$ is increasing in $c_{t}^{T}$.

The denominator and the third term in the numerator of condition (29) show the effects of the intermediation externality, because they show how the planner responds to the three effects of liability dollarization discussed earlier. The denominator reduces the social marginal utility of wealth (since $b_{t}^{c}<0$, it follows that $1-p^{c l}(t) b_{t}^{c}>1$ ), because the planner internalizes that an increase in $p^{c}(t)$ increases the burden of repaying outstanding debt, which reduces resources available for tradables consumption. The third term in the numerator increases the marginal utility of wealth (i.e. $\left.-p^{c}(t) b_{t}^{c}\left[\mathbb{E}_{t-1}\left(\lambda_{t}\right)+\frac{\operatorname{Cov}_{t-1}\left(\lambda_{t}, p^{c}(t)\right)}{\mathbb{E}_{t-1}\left[p^{c}(t)\right]}\right]>0\right) .{ }^{14}$ This occurs because the planner internalizes the intermediaries' no-arbitrage condition, and hence takes into account how the marginal utility of wealth responds to the effect of changes in date-t consumption prices on price expectations, the price of debt and incentives to borrow at $t-1$. In particular, the planner takes into account how these effects alter the amount of resources in units of tradables that debt contracted at $t-1$ generates and the social valuation of the risk-taking incentive at $t-1$. The term $\mathbb{E}_{t-1}\left(\lambda_{t}\right)$ reflects the effect of changes in the amount of resources that the debt contracted at $t-1$ can generate, which alter the marginal utility of wealth expected for date $t$. The term $\operatorname{Cov}_{t-1}\left(\lambda_{t}, p^{c}(t)\right) / \mathbb{E}_{t-1}\left[p^{c}(t)\right]$ has a similar form as the private risk-taking incentive identified earlier in condition (18), but in condition (29) the planner is internalizing its impact on the date- $t$ social marginal utility of wealth. The private incentive can be expressed as $\beta \frac{\operatorname{Cov}_{t-1}\left(p^{c}(t), \lambda\right)}{\mathbb{E}_{t-1}\left[p^{c}(t) \mid q_{t-1}^{*}\right.}$, and as explained earlier it reduces the expected marginal cost of borrowing between $t$ and $t+1$. The planner, in contrast, considers how this incentive alters the debt chosen at $t-1$ and thereby the debt repayment burden of date $t$.

[^10]It is important to note that the effects of the intermediation externality on the planner's marginal utility of wealth are present regardless of whether the collateral constraint binds, as long as we maintain the assumptions of incomplete markets and uncertainty. Under perfect foresight, the covariance term in condition (29) vanishes and the terms with $p^{c \prime}(t) b_{t}^{c}$ in the numerator and denominator cancel each other (since $\mathbb{E}_{t-1}\left[\lambda_{t}\right]=\lambda_{t}$ ). ${ }^{15}$ The same happens under complete markets because $\lambda_{t}$ becomes time- and state-invariant.

The intermediation externality also has the key implication that it makes the optimal policy under commitment time-inconsistent. This is evident because due to this externality the social marginal utility of wealth at date $t$ depends in part on how the planner's choice of consumption and debt at date t affects price expectations and the risk-taking-incentive at date $t-1$, which alter $\lambda_{t}$ by affecting the debt chosen at $t-1$ and hence the burden of debt repayment at date $t$. Notice time-inconsistency emerges in this model even though the collateral constraint is defined in terms of a limit on the debt-to-income ratio, or as a flow constraint, instead of a debt-to-assets ratio, or stock constraint. Hence, time-inconsistency of optimal financial policy under commitment can exist in Fisherian models with either stock or flow collateral constraints, what is necessary is for there to be a vehicle that allows the planner to affect past prices or borrowing capacity with current consumption or debt choices.

This time-inconsistency result is similar to Bianchi and Mendoza (2017)'s result showing the time-inconsistency of optimal macroprudential policy under commitment in a model in which assets serve as collateral, but the mechanism driving the time-inconsistency is different. In Bianchi and Mendoza, the asset-pricing condition connecting current asset prices to future consumption leads the planner to prop up asset prices when the collateral constraint binds by pledging that future consumption will be lower, which is not optimal to do ex-post. In contrast, in our setting, $p_{t}^{N}$ and $c_{t}^{T}$ are independent of the planner's date- $t$ choices when the constraint binds, so the planner cannot prop up the value of collateral by affecting $p_{t}^{N}$ when the constraint binds. ${ }^{16}$ Instead, the

[^11]planner can pledge higher consumption at $t+1$ so that higher expected prices prop up $q_{t}^{c}$ and reduce the ex-ante real interest rate, strengthening borrowing incentives and borrowing capacity via the intermediation-externality effects discussed earlier, and in particular by increasing the effective value of collateral as the price of bonds falls. Ex-post, however, delivering on this pledge is suboptimal, because higher prices at $t+1$ imply a higher ex-post real interest rate, and thus a higher burden of debt repayment in that period. This time-inconsistency mechanism is at work regardless of whether the constraint binds, but by affecting borrowing incentives it can affect the likelihood that the constraint can bind at equilibrium.

The effects of the macroprudential and intermediation externalities on borrowing decisions can be analyzed by comparing the optimality conditions equating the marginal costs and benefits of borrowing for the social planner and private agents. Combining conditions (26) and (27) with the result in (29), the planner's Euler equation for bonds can be rewritten as follows:

$$
\begin{align*}
u_{T}(t)=\beta \mathbb{E}_{t}\left[\left[u_{T}(t+1)+\mu_{t+1} \kappa y_{t+1}^{N} p^{N \prime}(t+1)\right] \tilde{R}_{t+1}^{c} \Psi(t\right. & +1)] \\
& +\mu_{t}\left(1-\psi(t)-\kappa y_{t}^{N} p^{N^{\prime}}(t)\right) \tag{30}
\end{align*}
$$

where:

$$
\begin{gather*}
\Psi(t+1) \equiv\left(\frac{1-\psi(t)}{1-\psi(t+1)}\right)  \tag{31}\\
\psi(t) \equiv p^{c \prime}(t) b_{t}^{c}\left[1-\left(\frac{\mathbb{E}_{t-1}\left[\lambda_{t}\right]}{\lambda_{t}}+\frac{\operatorname{Cov}_{t-1}\left(\lambda_{t}, p^{c}(t)\right)}{\lambda_{t} \mathbb{E}_{t-1}\left[p^{c}(t)\right]}\right)\right] \tag{32}
\end{gather*}
$$

Since using (32) we can rewrite (29) as $\lambda_{t}=\frac{u_{T}(t)+\mu_{\epsilon} \kappa p^{N^{\prime}}(t) y_{t}^{N}}{1-\psi(t)}$, it follows from the properties that $\lambda_{t}>0$ and $u_{T}(t)+\mu_{t} \kappa p^{N \prime}(t) y_{t}^{N}>0$ that $\psi(t)<1$. The sign of $\psi(t)$, however, cannot be determined analytically, because it involves effects of the intermediation externality that move $\lambda_{t}$ in opposite directions, as we explained earlier. The first term inside the square brackets in the right-hand-side of (32) reduces $\lambda_{t}$ via the direct effect of higher consumption prices on the burden of outstanding debt repayment, while the expectation and covariance terms increase $\lambda_{t}$ by affecting debt prices and borrowing incentives at $t-1$ and hence indirectly the date-t debt repayment burden. Since $b_{t}^{c}<0$, if the first (second) effect dominates, $\psi(t)$ is negative (positive). Moreover, since under rational expectations $\lambda_{t}=\mathbb{E}_{t-1} \lambda_{t} \pm \epsilon_{t}^{\lambda}$, where $\epsilon_{t}^{\lambda}$ is a white-noise error, it must be that on average
or for small $\epsilon_{t}, \psi(t) \approx p^{c \prime}(t) b_{t}^{c}\left[\frac{-\operatorname{Cov}_{t-1}\left(\lambda_{t}, p^{c}(t)\right)}{\lambda_{t} \mathbb{E}_{t-1}\left[p^{c}(t)\right]}\right]$, and thus the sign of $\psi(t)$ is the same as the sign of the planner's rist-taking-incentive conditional covariance. If this covariance is positive (negative), $0<\psi(t)<1(\psi(t)<0)$.

We can now compare the Euler equation of the planner with that of private agents in the decentralized equilibrium. To facilitate the comparison, combine conditions (7) and (9) to express the Euler equation of private agents as follows:

$$
\begin{equation*}
u_{T}(t)=\beta \mathbb{E}_{t}\left[\tilde{R}_{t+1}^{c} u_{T}(t+1)\right]+\mu_{t}^{D E} \tag{33}
\end{equation*}
$$

where we use $\mu_{t}^{D E}$ to distinguish the Lagrange multiplier of the collateral constraint in the decentralized equilibrium from the one corresponding to the planner's problem.

Consider first the terms including $\mu_{t}$ and $\mu_{t}^{D E}$. These terms are only present if the collateral constraint binds, but in this case the values of these multipliers are irrelevant for allocations. This is because, as we noted earlier, when the constraint binds at $t$, the resource and collateral constraints imply that the equilibrium allocation of $c_{t}^{T}$ is a value that satisfies condition (16), and this value is independent of $\mu_{t}$ and $\mu_{t}^{D E}$. Note, however, that allocations and prices will differ for the planner and the unregulated equilibrium, because by internalizing the externalities the planner will generally arrive at states in which the constraint binds with different $b_{t}^{c}$, which implies different allocations and prices.

Compare next conditions (30) and (33) when the collateral constraint does not bind. The macroprudential externality has the same overborrowing effect typical of SS models: The social planner's marginal cost for the debt chosen at date $t$ adds to $u_{T}(t+1)$ the term $\mu_{t+1} \kappa y_{t}^{N} p^{N^{\prime}}(t+1)$, because the planner internalizes that if the collateral constraint binds at $t+1$ the value of collateral falls and reduces borrowing capacity. This effect is always positive for any state in which $\mu_{t+1}>$ 0 , because $p^{N}$ is increasing in $c^{T}$. Hence, the macroprudential externality is an overborrowing externality, because it implies that the marginal cost of borrowing for private agents is lower than the social marginal cost.

The effect of the intermediation externality can be seen in the fact that in right-hand-side of
the planner's Euler equation the ex-post real interest rate is distorted by the term $\Psi(t+1)$, which is a ratio of this externality's effects on repayment debt burden, price of newly issued debt, and risk-taking incentive at $t$ relative to $t+1$. If $\Psi(t+1)$ is greater (smaller) than 1 , the intermediation externality increases (reduces) the social marginal cost of borrowing relative to the private marginal cost of borrowing. The size of this ratio cannot be determined unambiguously, but using condition (32) we can infer that, everything else constant, $\Psi(t+1)$ is greater (smaller) than 1 if the planner's real-exchange-rate expectations (i.e. the ratio $\left.\mathbb{E}_{t}\left[p^{c}(t+1)\right] / \mathbb{E}_{t-1}\left[p^{c}(t)\right]\right)$ are increasing (decreasing) sufficiently. Hence, when the planner expects a sufficiently strong real appreciation, it faces an effective ex-post real interest rate higher than the one faced by private agents, and the intermediation externality operates as a second overborrowing externality. The opposite occurs when a sufficiently strong real depreciation is expected. Moreover, unlike the macroprudential externality that is only present when $\mu_{t+1}$ is expected to be positive, the intermediation externality is always present, regardless of whether the constraint is expected to bind or not (although the actual value of $\Psi(t+1)$ does depend on whether the constraint is expected to bind).

The optimal allocations under commitment can be decentralized with a debt tax. Since allocations and prices are independent of $\mu_{t}$ when the constraint binds at $t$, the relevant use of the debt tax is when $\mu_{t}=0$. In this case, the optimal debt tax is the one that equalizes the social and private marginal costs of borrowing given by the right-hand-sides of conditions (30) and (33) with $\mu_{t}=0:$

$$
\begin{equation*}
\tau_{t}=\frac{\mathbb{E}_{t}\left[\left(u_{T}(t+1)+\mu_{t+1} \kappa y_{t+1}^{N} p^{N^{\prime}}(t+1)\right) \tilde{R}_{t+1}^{c} \Psi(t+1)\right]}{\mathbb{E}_{t}\left[\tilde{R}_{t+1}^{c} u_{T}(t+1)\right]}-1 \tag{34}
\end{equation*}
$$

The numerator of this expression includes terms that correspond to the macroprudential and intermediation externalities. Since the intermediation externality can yield social marginal costs of borrowing higher or lower than their private counterparts, in principle the tax could be negative (i.e. a subsidy). The tax also inherits the time-inconsistency of the social planner's optimal decisions.

If we switch to the standard SS model by imposing on the above result the assumption that debt is issued in units of tradables, the expression reduces to $\tau_{t}=\frac{\mathbb{E}_{t}\left[\mu_{t+1} \kappa y_{t_{+1}} p^{p^{\prime}}(t+1)\right]}{\mathbb{E}_{t}\left[u_{T}(t+1)\right]}$, which is the optimal debt tax of the standard SS model (e.g. Bianchi (2011), Bianchi et al. (2016)). The above optimal tax also preserves the result from the standard SS model that the value of
the tax is indeterminate when $\mu_{t}>0$, because as explained above the planner's allocations are independent of $\mu_{t}$ when the collateral constraint binds. Therefore, any debt tax consistent with the collateral constraint being binding in the decentralized equilibrium with debt taxes can support the planner's allocations when $\mu_{t}>0$. In quantitative applications, the convention in the literature is to determine if a zero debt tax is consistent with this outcome, and if so the tax is assumed to be zero when the constraint binds.

The planner does not require additional instruments to implement its optimal plans, but the same optimal plans can be decentralized with alternative instruments. In particular, we established earlier in condition (22) that a tax on capital inflows to financial intermediaries is equivalent to a debt tax on private agents. Hence, setting the debt tax on domestic borrowing by private agents to zero in all periods and a tax on bonds sold abroad by intermediaries to a rate $\theta_{t}$ that is zero if $\mu_{t}>0$ and equal to the expression in the right-hand-side of (34) otherwise supports identical allocations as the optimal debt tax. Thus, under commitment the SSLD model shares the property of the SS model that it does not justify the use of capital controls.

It is worth comparing the optimal debt tax in (34) with the one that would be used by a planner that can borrow directly from foreign lenders in units of tradables. The planner's problem would be identical to the one solved in the standard SS model (e.g. Bianchi (2011)). Hence, there would not be a time-inconsistency problem. But, since private agents still borrow from intermediaries in units of aggregate consumption, decentralizing the optimal allocations would not result in the same debt taxes as in the standard SS model. In particular, the optimal debt tax is:

$$
\begin{equation*}
\tau_{t}=\frac{1+\frac{\mathbb{E}_{t}\left[\mu_{t+1} \kappa y_{t+1}^{N} p^{N^{\prime}}(t+1)\right]}{\left.\mathbb{E}_{t} u_{T}(t+1)\right]}}{1+\frac{\operatorname{Cov} t\left(u_{T}(t+1), p^{c}(t+1)\right)}{\mathbb{E}_{t}\left[u_{T}(t+1)\right] \mathbb{E}_{t}\left[p^{c}(t+1)\right]}}-1 \tag{35}
\end{equation*}
$$

The numerator in the first term in the right-hand-side of this expression is identical to the optimal macroprudential debt tax of the standard SS model, but the term in the denominator is new. This term is related to the risk-taking incentive component affecting the Euler equation of bond holdings in the decentralized equilibrium without regulation. Since the covariance in this term is negative, and its ratio divided by $\mathbb{E}_{t}\left[u_{T}(t+1)\right] \mathbb{E}_{t}\left[p^{c}(t+1)\right]$ is a positive fraction in absolute value, the optimal debt tax in this model is always higher than in the standard model, and there is a positive
tax even in states in which $\mathbb{E}_{t}\left[\mu_{t+1}\right]=0$. Hence, this planner's optimal tax is still correcting for an intermeditaion externality that results from liability dollarization, but the externality reduces to only the risk-taking-incentive term.

### 3.2 Conditionally Efficient Planner

The time-inconsistency of the optimal financial policy under commitment is a serious drawback, because it implies that the policy is not credible. Bianchi and Mendoza (2010) and (2017) proposed two different approaches to tackle this issue: First, specifying a conditionally efficient regulator committed to support the asset pricing function of the unregulated decentralized equilibrium. Second, solving for optimal policy without commitment as a Markov stationary equilibrium in which the planner as of date $t$ formulates optimal plans taking as given conjectures of the optimal plans of the planner making decisions at $t+1$. Here, we follow the first approach by assuming that the planner is committed to support the pricing function of domestic debt $q^{c}$ from the unregulated decentralized equilibrium.

The recursive formulation of the conditionally efficient SP's problem is:

$$
\begin{align*}
& V\left(b^{c}, y^{T}, y^{N}\right)=\max _{\left\{b^{c \prime}, c^{T}\right\}}\left[u\left(c\left(c^{T}, y^{N}\right)\right)+\beta \mathbb{E}_{\left(y^{T^{\prime}}, y^{N \prime}\right) \mid\left(y^{T}, y^{N}\right)}\left[V\left(b^{c \prime}, y^{T \prime}, y^{N \prime}\right)\right]\right]  \tag{36}\\
& \text { s.t. } \\
& q^{\mathrm{DE}}\left(b^{c}, y^{T}, y^{N}\right) p^{c}\left(c^{T}, y^{N}\right) b^{c \prime}+c^{T}=p^{c}\left(c^{T}, y^{N}\right) b^{c}+y^{T}  \tag{37}\\
& q^{\mathrm{DE}}\left(b^{c}, y^{T}, y^{N}\right) p^{c}\left(c^{T}, y^{N}\right) b^{c \prime} \geq-\kappa\left(y^{T}+p^{N}\left(c^{T}, y^{N}\right) y^{N}\right) \tag{38}
\end{align*}
$$

where $q^{\mathrm{DE}}\left(b^{c}, y^{T}, y^{N}\right)$ is the recursive equilibrium pricing function of domestic bonds in the decentralized equilibrium without regulation.

Deriving the first-order conditions for the above problem and using the envelope theorem yields
the following optimality conditions in sequential form:

$$
\begin{align*}
& \lambda_{t}=\frac{u_{T}(t)+\mu_{t} \kappa p^{N^{\prime}}(t) y_{t}^{N}-p^{c \prime}(t) q^{\mathrm{DE}}(t) b_{t+1}^{c}\left(\lambda_{t}-\mu_{t}\right)}{1-p^{c \prime}(t) b_{t}^{c}}  \tag{39}\\
& \lambda_{t}=\beta \mathbb{E}_{\left(y^{T \prime}, y^{N^{\prime}}\right) \mid\left(y^{T}, y^{N}\right)}\left[\tilde{R}_{t+1}^{c}\left[\lambda_{t+1}-\left(\lambda_{t+1}-\mu_{t+1}\right) q^{\mathrm{DE} \prime}(t+1) b_{t+2}^{c}\right]\right]+\mu_{t}  \tag{40}\\
& q^{\mathrm{DE}}(t) p^{c}(t) b_{t+1}^{c}+\kappa\left(y_{t}^{T}+p^{N}(t) y_{t}^{N}\right) \geq 0, \quad \text { with equality if } \mu_{t}>0 \tag{41}
\end{align*}
$$

where $q^{\mathrm{DE}^{\prime}}(t+1) \equiv \frac{\partial q^{D E}\left(b_{t+1}^{c}, y_{t+1}^{T}, y_{t+1}^{N}\right)}{\partial b_{t+1}^{c}}$.
The marginal utility of wealth in condition (39) is similar to the comparable expression for the planner with commitment (condition (26)). The first two terms in the numerator and the term in the denominator are identical, and have the same interpretation. Both planners take into account the effect of changes in the price of nontradables on borrowing capacity, and both planners internalize that higher $c_{t}^{T}$ increases the burden of repaying outstanding debt because it increases $p^{c}(t)$, and hence reduces $\lambda_{t}$. The key difference between the two planners is in the third term in the numerators of (39) v. (26). Both terms are positive, because we are focusing on states with $b^{c}<0$ and $\lambda_{t}>\mu_{t}$, but the timing of the effects they capture differs. ${ }^{17}$ The conditionally efficient planer internalizes that an increase in $p^{c}(t)$ increases the value of the new debt issued at date $t$ in units of tradables by the amount $-p^{c^{\prime}}(t) q^{D E}(t) b_{t+1}^{c}$, and therefore $\lambda_{t}$ increases. In contrast, the planner with commitment internalizes that an increase in $p^{c}(t)$ increases the value of the new debt issued at date $t-1$ in units of tradables, because it affects the price at which $b_{t}^{c}$ was issued (i.e. $q_{t-1}^{c}$ ) via the effects on the intermediaries' no-arbitrage condition discussed earlier, which in turn increase $\lambda_{t}$. Time-inconsistency disappears from the conditionally efficient planner's problem because the effect by which the current choice of $c_{t}^{T}$ affects the value of debt chosen at $t-1$ under commitment is replaced by an effect that affects the value of debt chosen at $t$.

[^12]Using equations (39) and (40) we can rewrite the Euler equation of the planner as:

$$
\begin{array}{r}
u_{T}(t)=\beta \mathbb{E}_{t}\left[\left[u_{T}(t+1)+\mu_{t+1} \kappa y_{t+1}^{N} p^{N \prime}(t+1)\right] \tilde{R}_{t+1}^{c} \hat{\Psi}(t+1) \Omega(t+1)\right]  \tag{42}\\
+\mu_{t}\left(1-\hat{\psi}(t)-\kappa y_{t}^{N} p^{N \prime}(t)\right)
\end{array}
$$

where:

$$
\begin{array}{r}
\hat{\psi}(t) \equiv p^{c^{\prime}}(t)\left[b_{t}^{c}-q^{\mathrm{DE}}(t) b_{t+1}^{c}\left(1-\frac{\mu_{t}}{\lambda_{t}}\right)\right] \\
\hat{\Psi}(t+1) \equiv\left(\frac{1-\hat{\psi}(t)}{1-\hat{\psi}(t+1)}\right) \\
\Omega(t+1) \equiv\left(1-\left(1-\frac{\mu_{t+1}}{\lambda_{t+1}}\right) q^{D E^{\prime}}(t+1) b_{t+2}^{c}\right) \tag{45}
\end{array}
$$

As before, the effects of the macroprudential and intermediation externalities on borrowing decisions can be inferred by comparing conditions (42) and (33) when $\mu_{t}=0$. Once again, the macroprudential externality has the same overborrowing effect typical of SS models given by the term $\mu_{t+1} \kappa y_{t}^{N} p^{N^{\prime}}(t+1)$, because the planner internalizes that the value of collateral falls and reduces borrowing capacity in states with $\mu_{t+1}>0$. Hence, on account of the macroprudential externality, the private marginal cost of borrowing is lower than the social marginal cost.

The intermediation externality operates via two wedges. First, the wedge $\hat{\Psi}(t+1)$ is similar to the wedge identified in the case of the planner with commitment. The differences in the expressions for $\hat{\Psi}(t+1))$ and $\hat{\psi}(t)$ relative to those obtained for the planner with commitment, $\Psi(t)$ and $\psi(t)$ in equations (31) and (32) respectively, are due to the timing differences noted earlier in the determinants of $\lambda_{t}$ for each planner. ${ }^{18}$ As in the case with commitment, whether this wedge reduces or increases the social marginal cost of borrowing relative to the private one (i.e. whether it induces underborrowing or overborrowing) depends on whether $\hat{\Psi}(t+1)$ is greater or smaller than 1 , which in turn depends on the relative magnitudes of the intermediation externalities at dates $t \mathrm{v} . t+1$. We can also infer that, everything else constant, $\hat{\Psi}(t+1)$ is greater (smaller) than 1 if the planner

[^13]expects a relatively larger real appreciation between $t$ and $t+1$ than between $t+1$ and $t+2$ (i.e. if $\frac{\mathbb{E}_{t}\left[p_{t+1}^{c}\right]}{p_{t}^{+}}$is sufficiently larger than $\frac{\mathbb{E}_{t+1}\left[p_{t+2}^{c}\right]}{p_{t+1}^{c}}$ ). This result is qualitatively similar to the one obtained under commitment, in the sense that if $\mathbb{E}_{t}\left[p_{t+1}^{c}\right]$ is sufficiently high the intermediation externality causes the effective ex-post real interest rate to be higher than $\tilde{R}_{t+1}^{c}$, and thus operates as an overborrowing incentive by increasing the social marginal costs of borrowing above the private cost.

The second wedge is $\Omega(t+1)$. This wedge is always smaller than 1 for $b_{t+2}^{c}<0$ and is present because the conditionally efficient planner internalizes the effect of the debt chosen at date $t$ on the price of the debt that will be contracted at $t+1$, as reflected by the term $q^{D E \prime}(t+1)$ in eq. (45). ${ }^{19}$ Higher debt (lower $b_{t+1}^{c}$ ) increases $q^{D E}$ making borrowing at $t+1$ less costly. Hence, this is an underborrowing externality, because on account of $\Omega(t+1)<1$ the social marginal cost of borrowing is lower than the private marginal cost. As before, the two wedges that represent the intermediation externality are present regardless of whether the credit constraint is expected to bind or not, although the magnitude of the two wedges does depend on whether the constraint is expected to bind.

Decentralizing the optimal allocations of the conditionally efficient planner requires two instruments, debt taxes and capital controls. Since we showed using condition (22) that a debt tax $\tau_{t}$ combined with a tax on capital inflows $\theta_{t}$ yields an effective tax on debt at a rate of $\left(1+\tau_{t}^{e f}\right) \equiv\left(1+\tau_{t}\right)\left(1+\theta_{t}\right)$, we construct the optimal policies by first determining the optimal effective debt tax, then using the financial intermediaries' no-arbitrage condition we determine $\theta_{t}$, and then determine $\tau_{t}$ as the value implied by $\theta_{t}$ and $\tau_{t}^{e f}$.

The optimal $\tau^{e f}$ equates the private and social marginal costs of borrowing. As in the decentralized equilibrium without regulation, allocations when $\mu_{t}>0$ are independent of the value of $\mu_{t}$, and hence any $\tau^{e f}$ consistent with the constraint binding would support them. When $\mu_{t}=0$,

[^14]the optimal $\tau_{t}^{e f}$ that equates the right-hand-side of conditions (42) and (33) is:
\[

$$
\begin{equation*}
\tau_{t}^{e f}=\frac{\mathbb{E}_{t}\left[\left(u_{T}(t+1)+\mu_{t+1} \kappa y_{t+1}^{N} p^{N^{\prime}}(t+1)\right) \tilde{R}_{t+1}^{c} \hat{\Psi}(t+1) \Omega(t+1)\right]}{\mathbb{E}_{t}\left[\tilde{R}_{t+1}^{c} u_{T}(t+1)\right]}-1 \tag{46}
\end{equation*}
$$

\]

The optimal $\theta_{t}$ is needed so as to guarantee that the planner's allocations support the same pricing function for domestic debt as in the unregulated decentralized equilibrium. Intuitively, since supporting the same pricing function is what rules out time-inconsistency in this planner's problem, capital controls are being used to make the optimal policies credible. To accomplish this, the planner needs to impose capital controls at a rate $\theta_{t}$ such that:

$$
\begin{equation*}
\theta_{t}=\frac{q_{t}^{*}}{q^{\mathrm{DE}}(t)} \frac{\mathbb{E}_{t}\left[p^{c}(t+1)\right]}{p^{c}(t)}-1 \tag{47}
\end{equation*}
$$

This condition guarantees that the no-arbitrage condition of the intermediaries in the regulated competitive equilibrium supports the equilibrium pricing function $q^{D E}(t)$ of the unregulated competitive equilibrium but at the values of $\mathbb{E}_{t}\left[p^{c}(t+1)\right]$ and $p^{c}(t)$ determined by the allocations chosen by the planner. Notice that this also implies that the optimal tax on capital inflows equals the ratio of the ex-ante real interest rates in the unregulated decentralized equilibrium relative to the one implied by the planner's allocations. Capital controls are tighter the higher the ex-ante real interest rate in the absence of regulation relative to the socially optimal one. Moreover, $\theta_{t}$ rises (falls) when the real exchange is expected to appreciate (depreciate), and since the real exchange rate is increasing in $c^{T}$, this also implies that the capital controls are tighter the higher is the expected growth rate of tradables consumption.

While the values of $\tau_{t}^{e f}$ and $\tau_{t}$ are undetermined when the collateral constraint bind, in the sense that they can be set to any value such that $\mu_{t}>0, \theta_{t}$ still has a well-defined value. This is because, in order to support the same pricing function of domestic debt as in the decentralized equilibrium without regulation, a particular value of $\theta_{t}$ is needed for condition (47) to hold even when $\mu_{t}>0$. Intuitively, capital controls are adjusting as needed so that debt pricing cannot be affected by the planner's choices, thereby supporting the time consistency of the conditionally-efficient planner's choices.

Given $\tau_{t}^{e f}$ and $\theta_{t}$, the debt tax rate that satisfies $\left(1+\tau_{t}^{e f}\right) \equiv\left(1+\tau_{t}\right)\left(1+\theta_{t}\right)$ is:

$$
\begin{equation*}
\tau_{t}=\frac{\mathbb{E}_{t}\left[\left(u_{T}(t+1)+\mu_{t+1} \kappa y_{t+1}^{N} p^{N \prime}(t+1)\right) \tilde{R}_{t+1}^{c} \tilde{\Psi}(t+1) \Omega(t+1)\right]}{\mathbb{E}_{t}\left[\tilde{R}_{t+1}^{c} u_{T}(t+1)\right]} \frac{q^{\mathrm{DE}}(t) p^{c}(t)}{q_{t}^{*} \mathbb{E}_{t}\left[p^{c}(t+1)\right]}-1 \tag{48}
\end{equation*}
$$

Hence, in the case of the conditionally efficient planner, implementing the SP allocations requires both domestic debt taxes and capital controls. The debt taxes are used to correct the macroprudential and intermediation externalities, and the capital controls are used so that the planner cannot affect the debt pricing function (i.e. so that its optimal plans remain time-consistent). Capital controls are therefore used as an instrument to preserve the credibility of the optimal policy.

As in Bianchi (2011), we can construct "exact" values of $\tau^{e f}, \tau$ that not only support the SP's allocations but that also equate the private and social shadow values of relaxing the credit constraint when it binds. To solve for these, we first solve for the value of $\tau^{e f}$ that equates conditions (42) and (33) even when $\mu_{t}>0$. Then, given the optimal $\theta_{t}$, we use again $\left(1+\tau_{t}^{e f}\right) \equiv\left(1+\tau_{t}\right)\left(1+\theta_{t}\right)$ and the exact value of $\tau^{\text {ef }}$ to solve for the exact value of $\tau$ :

$$
\begin{equation*}
\tau_{t}=\frac{\beta \mathbb{E}_{t}\left[\left(u_{T}(t+1)+\mu_{t+1} \kappa y_{t+1}^{N} p^{N \prime}(t+1)\right) \tilde{R}_{t+1}^{c} \tilde{\Psi}(t+1) \Omega(t+1)\right]-\mu_{t}\left(\hat{\psi}(t)+\kappa y_{t}^{N} p^{N \prime}(t)\right)}{\beta \mathbb{E}_{t}\left[\tilde{R}_{t+1}^{c} u_{T}(t+1)\right]\left(1+\theta_{t}\right)}-1 \tag{49}
\end{equation*}
$$

## 4 Quantitative Analysis

### 4.1 Calibration

The baseline calibration of the model's parameters is the same as in Bianchi (2011), which was constructed to match data for Argentina at an annual frequency. The only two differences are that we keep the nontradables endowment constant at $y^{N}=1$ to reduce the dimensionality of the state space, and for simplicity we apply the same value of $\kappa$ to tradables and nontradables collateral. The parameter values are listed in Table 1.

Table 1: Baseline Calibration

| Parameter | Value |
| :---: | :---: |
| $\gamma$ | 2 |
| $\eta$ | 0.205 |
| $\omega$ | 0.31 |
| $\beta$ | 0.91 |
| $q^{*}$ | 0.96 |
| $\rho_{y^{T}}$ | 0.54 |
| $\sigma_{y^{T}}$ | 0.059 |
| $y^{N}$ | 1.00 |
| $\kappa$ | 0.32 |

Bianchi (2011) set the coefficient of relative risk aversion to $\gamma=2$, which is a standard value. He also set $\eta=0.205$, so that the elasticity of substitution between tradables and nontradables $(1 /(1+\eta))$ is 0.83 , which is the upper bound of a range of existing estimates. This elasticity is key for determining the elasticities of both $p^{c}$ and $p^{N}$ to changes in sectoral consumption allocations, both of which play a central role in the pecuniary and intermediation externalities and in the price responses when a Sudden Stop occurs.

The Markov process for the tradable endowment $y^{T}$ is designed to match an $\operatorname{AR}(1)$ time-series process discretized using Tauchen and Hussey (1991)'s approach. We allow for 9 realizations of the endowment (with $\mathbb{E}\left[y^{T}\right]=1$ ), setting the autocorrelation coefficient $\rho_{y^{T}}$ to 0.54 and the standard deviation to $\sigma_{y^{T}}=0.059$, which are Bianchi's estimates for an autoregressive process of the cyclical component of tradables GDP in Argentina.

The discount factor $\beta$ is set to match Argentina's average net foreign asset position-GDP ratio of -0.29 , which is computed using the data reported by Lane and Milesi-Ferretti (2001). $\omega=0.31$ is set so as to match a tradable consumption share of 32 percent. The world's risk-free interest rate is set to $4 \%$ (i.e. $q^{*}=0.96$ ), which is a standard number in DSGE models. We also set the value of $\kappa=0.32$ to be the same as in Bianchi (2011), but notice that in the SSLD model it yields
a probability of crisis of $3 \%$, which is in line with empirical estimates but lower than in Bianchi's SS model (5.5\%).

### 4.2 Decision Rules, Amplification, \& Crisis Dynamics: SSLD v. SS Model

We solve the decentralized equilibrium (DE) without policy intervention of the SS and SSLD models in recursive form using a variant of a time-iteration algorithm that takes into account the occasionally binding collateral constraint. Further details are provided in the Appendix. We start the analysis of the results by comparing the borrowing decisions of the two models. Figure 2 presents the debt decision rules, as well as the corresponding credit limits $\left(\bar{B}_{S S}^{c}(b, y)\right.$ and $\bar{B}_{S S L D}^{c}(b, y)$, respectively), evaluated at the average value of $y^{T} .{ }^{20}$ Since debt is denominated in units of tradables in the SS model, we make the debt decision rule and debt limits of the SSLD model comparable by plotting them also in units of tradables (i.e. multiplied by $p^{c}$ ).

Figure 2: Debt Decision Rules in the Decentralized Equilibrium: SS and SSLD Models


As in Bianchi (2011), we divide the Figure into three regions: The Constrained Credit Region is located where the collateral constraint is binding for the SSLD model. The Positive Crisis

[^15]Probability Region is where the collateral constraint is not binding at $t$ but can bind with positive probability at $t+1$ in some states of nature for the SSLD model. The Stable Credit Region is where the collateral constraint does not bind at $t$ and is not expected to bind in any state of nature at $t+1$ for the SSLD model.

The debt decision rule in the SS model is very similar to the one in Bianchi (2011), which is natural since we are using a nearly identical calibration. The downward sloping segments of the two decision rules reflect the severity of the Fisherian debt deflation mechanism in both models: When the collateral constraint binds, the drop in $p^{N}$ deflates the value of collateral and produces an endogenous reduction in access to credit, which turns the decision rules for new bond holdings into decreasing functions of outstanding bond holdings (i.e. the higher the outstanding debt the more severe the deflation and the larger the reversal in the new debt position). In this regard, the SS and SSLD models are similar, because the slopes of the two decision rules are not very different, even though in the SSLD model there are also endogenous changes in $p^{c}$ and $q^{c}$ affecting borrowing capacity when the constraint binds.

Two important results are evident in Figure 2: (i) debt positions, namely the stocks $p^{c} b^{\prime c}$, in the SSLD model are larger in all three regions (the debt decision rule of the SSLD model is always below that of the SS model); (ii) borrowing capacity is larger in the SSLD model (the credit limit of the SSLD model is always below that of the SS model). There are three important additional results that also emerge from Figure 2 by studying it in more detail: (iii) credit flows or current account deficits, which are given by $C A \equiv p^{c} b^{c}-p^{c} b^{c}$, are much larger in the SSLD model (for $p^{c} b^{c}>0.87$ the SSLD model has a larger deficit and in the rest of the Positive Crisis Probability Region the SS model is in surplus while the SSLD remains in deficit); (iv) the collateral constraint starts binding at slightly lower debt levels (higher $p^{c} b^{c}$ ) in the SSLD model; and (v) in the Constrained Credit Region, reversals in debt positions and the current account are smaller in the SSLD model than in the SS model for the same value of initial bond holdings. ${ }^{21}$ These results indicate that, under the baseline calibration, the incentives for additional borrowing induced by the intermediation externality dominate, and hence liability dollarization expands borrowing capacity,

[^16]increases debt positions and current account deficits, and yields smaller corrections in debt and the current account when credit is constrained at the same value of initial bond holdings. This last result suggests that the stochastic SSLD model may retain the result from the deterministic variant indicating that Sudden Stops are less severe than in the SS model along the equilibrium path.

Figure 3 sheds light on the implications of the differences in debt decision rules for consumption of tradables. The Figure shows decision rules for $c^{T}$ in the SS and SSLD models for average $y^{T}$, with the state space separated into the same three regions as in the previous Figure.

Figure 3: Decision Rules for $c^{T}$ in the Decentralized Equilibrium: SS and SSLD Models


Figure 3 shows that tradables consumption is higher in the SSLD model in all three regions, with the difference growing larger as initial debt rises ( $p^{c} b^{c}$ falls) and much larger when the credit constraint binds. These results are consistent with the findings from the debt decision rules showing larger debt positions and credit flows in the SSLD model, and smaller reversals in the Constrained Credit Region. As noted above, the slopes of the debt decision rules are similar across the two models in this region, but yet the $c^{T}$ decision rule of the SS model is significantly steeper. This is because, although the Fisherian deflation effect (as measured by the slope of the debt decision rules) is similar, the SS model experiences larger current account reversals in that region. As a result, declines in consumption during Sudden Stop events are likely to be smaller under liability
dollarization. At this point, however, this is only a conjecture, because the decision rules show only the equilibrium choices for any given state of nature $\left(p^{c} b^{c}, y^{T}\right)$. To compare the characteristics of macroeconomic aggregates along the equilibrium path of each model, we need to compare their ergodic distributions and equilibrium time-series dynamics, as we do next.

We construct ergodic distributions and Sudden Stop event windows using simulated datasets obtained by simulating the SS and SSLD models for 100,000 periods, truncating the first 1,000 to remove dependence of initial conditions. Figure 4 presents the long-run distributions of bond holdings in units of tradables. This Figure shows that, as the comparison of the debt decision rules suggested, liability dollarization sustains significantly larger debt positions at equilibrium. In fact, the majority of the debt positions with positive long-run probability in the SSLD model (i.e. $\left.p^{c} b^{c} \leq-0.9\right)$ have zero long-run probability in the SS model.

Figure 4: Long-Run Distributions of Debt: SS $\left(b^{c}\right)$ v. SSLD $\left(p^{c} b^{c}\right)$ Models


The SSLD model's simulated dataset and ergodic distribution can also be used to construct an estimate of the risk-taking borrowing incentive discussed in Section 2. To this end, we first rewrite
equation (18) as:

$$
\begin{equation*}
u_{T}(t)=\beta \mathbb{E}_{t}\left[u_{T}(t+1)\right]\left(R_{t+1}^{*}+\frac{\operatorname{Cov}_{t}\left(u_{T}(t+1), \tilde{R}_{t+1}^{c}\right)}{\mathbb{E}_{t}\left[u_{T}(t+1)\right]}\right)+\mu_{t} \tag{50}
\end{equation*}
$$

In this expression, the covariance term appears as a risk-premium-like term that is normalized by expected marginal utility, so that the risk-taking incentive is measured as an addition or subtraction to the world real interest rate. On average over the ergodic distribution of the SSLD model, $\frac{\operatorname{Cov}_{t}\left(u_{T}(t+1), \tilde{R}_{t+1}^{c}\right)}{\mathbb{E}_{t}\left[u_{T}(t+1)\right]}=-0.0046$. Hence, the risk-taking incentive is equivalent to an average reduction of 46 basis points in the world real interest rate (i.e. it is equivalent to lowering $R^{*}$ from $4 \%$ to $3.54 \%$, which is about a $12 \%$ reduction). This result can also be interpreted in terms of an implicit hedge that liability dollarization provides: Borrowing becomes more attractive because during downturns the price at which debt has to be repaid falls with consumption, which via the reduction in the burden of debt repayment provides an implicit form of insurance in bad times. This is absent from SS models, because debt is in units of tradables, which have a constant world-determined price, and hence debt repayment cannot co-move with domestic consumption.

We now compare Sudden Stop event dynamics in the SS and SSLD models by analyzing the simulated datasets following the same procedure as in event studies that characterize the empirical regularities of actual Sudden Stops (e.g. Korinek and Mendoza (2014)). We identify Sudden Stop events in each model's simulated dataset, defining Sudden Stops as periods in which the borrowing constraint binds and the current account-to-GDP ratio (CA/Y) raises by more than two standard deviations above its long-run average. Then we construct seven-year event windows for the key macroeconomic aggregates expressed in deviations from the long-run averages of each model economy, centered in the period in which Sudden Stops occur, and averaging across events within each date. Figure 5 shows event windows for a) aggregate consumption $\left(c_{t}\right)$, b) tradables consumption $\left.\left(c_{t}^{T}\right), \mathrm{c}\right)$ the price of nontradables $\left.\left(p_{t}^{N}\right), \mathrm{d}\right)$ the consumption price or real exchange rate $\left.\left(p_{t}^{c}\right), \mathrm{e}\right)$ the end-of-period bond position as a share of tradables output $\left.\left(p_{t}^{c} b_{t+1}^{c} / y_{t}^{T}\right), \mathrm{f}\right)$ the current account-GDP ratio $\left.\left(C A_{t} / Y_{t}\right), \mathrm{g}\right)$ the one-step-ahead conditional expectation of consumption prices or of the real exchange rate $\left.\left(\mathbb{E}_{t}\left[p_{t}^{c} b_{t+1}^{c}\right]\right), \mathrm{h}\right)$ the ex-ante debt price relative to the world price of bonds $\left(q_{t}^{c} / q^{*}\right)$ and i) the ex-post debt price also relative to the world price of bonds $\left(\tilde{q}_{t}^{c} / q^{*}\right) .{ }^{22}$ Notice that $p^{c}$ can

[^17]also be computed for the SS model, and then it can be used to compute the implied ex-ante and ex-post returns in units of $c$ of the bonds denominated in units $c^{T}$.

Figure 5: Sudden Stops in the SS and SSLD Models


Note: All variables except those measured as output ratios are plotted as percent deviations of their corresponding long-run averages. Variables measured as output ratios are shown as differences relative to the long-run average of the corresponding ratio and expressed in percent.

These event windows show that despite the larger debt positions and current account flows of the SSLD model, Sudden Stop events are less severe. In Sudden Stops (at $t=0$ ), aggregate and tradables consumption fall 4.5 and 14 percentage points below their long-run means respectively in the SSLD model, v. 13 and 35 percentage points in the SS model. The real exchange rate and the price of nontradables fall by 11 and 15 percentage points respectively in the SSLD model v. 30 and 40 percentage points in the SS model. The less severe Sudden Stops of the SSLD model are, however, more in line with the data. As documented by Korinek and Mendoza (2014), the cross-country average of declines in consumption and the real exchange rate relative to trend
during emerging markets Sudden Stops are about 3 and 5 percent respectively (the SSLD model predicts 4.5 and 11 percent, while the SS model predicts 14 and 30 percent). Similarly, the trade balance-output ratio increases by 10.5 percentage points when a Sudden Stop hits in the SS model v. 1.4 percentage points in the SSLD model, which is closer to the average reversal of 3.2 percentage points in the data.

The movements in ex-ante and ex-post debt prices (or interest rates) shown in panels h) and i) are explained by the dynamics of the actual v . expected real-exchange-rate movements shown in panels d) and g). To compare the ex-ante and ex-post debt prices, notice that since $q_{t}^{c} / q_{t}^{*}=R_{t+1}^{*} / R_{t+1}^{c}$ and $\tilde{q}_{t}^{c} / q_{t}^{*}=R_{t+1}^{*} / \tilde{R}_{t+1}^{c}$, the debt price ratios in the plots are spreads of the world real interest rate relative to the ex-ante and ex-post real interest rates respectively. Notice also that to compare the date- $t$ ex-ante rate with its appropriate ex-post counterpart, the ex-post rate shown for date $t$ is given by the ratio $\mathbb{E}_{t}\left[p_{t+1}^{c}\right] / p_{t+1}^{c}$, which has in the denominator the realized $p_{t+1}^{c}$ even though this is unknown at date t . This is so that in comparing date- $t$ rates we are comparing what the contracted ex-ante interest rate was expected to yield with the ex-post actual return that was actually obtained, both in units of tradables.

Ex-ante and ex-post bond prices do not differ that much across the two models, except when Sudden Stops hit, when the prices rise sharply but much less in the SSLD model (recall that ex-post prices shown for $t=-1$ are ex-post returns paid at $t=0$ on bonds issued at $t=-1)$. Ex-ante bond prices rise sharply because of the large real-exchange-rate collapses at $t=0$ (see Plot d )), and also because of higher expected prices for the next period (see Plot g)). Ex-post bond prices rise because, for debt issued at $t=-1$, real-exchange rates were much higher when that debt was issued than when it was repaid at $t=0$ (see Plot d). In terms of real interest rates, this means that both ex-ante and ex-post real interest rates fall sharply when Sudden Stops hit, but by less in the SSLD than in the SS model. Notice also that, within each model, ex-ante bond prices are higher than ex-post prices at $t=0$ (i.e. ex-ante real interest rates are lower), but they also differ in the other time periods. For the SSLD model, this illustrates that the effects of the intermediation externality are at work regardless of whether the collateral constraint binds or not. For the SS model, these are movements in the implied ex-ante and ex-post returns in units of $c$ of debt issued in units of $c^{T}$. These are only "notional" movements, in the sense that changes in ex-ante prices
do not affect the resources in tradable goods that debt generates, and changes in ex-post prices do not affect the burden of repayment of existing debt.

Plot e) shows that, as conjectured earlier, the two economies arrive at Sudden Stop states with different debt levels. The SSLD economy builds up more debt in general, as the ergodic distributions showed, but as a deviation of the mean debt- $y^{T}$ ratio, Plot e) shows that the debt is smaller in the SSLD than in the SS model at $t=0$ and in the three periods before. The debt ratios themselves are higher in the SSLD model, but as deviations from the mean they are smaller. This is also the case because the two economies also hit Sudden Stops at different income levels ( $y^{T}$ is $2.1 \%$ higher in the SSLD economy at $t=0$ ). Interestingly, in terms of the current account as a ratio of total GDP, the two economies display nearly the same ratio at $t=0$ and nearly the same reversal from $t=-1$. Relative to the data, these current account reversals are within the range of what is observed during Sudden Stops, albeit at the high end. Mendoza and Smith (2006) reported reversals in the current account-output ratio during the 1990s Sudden Stops ranging from 4 percentage points (Argentina, 1995) to 11 percentage points (Korea, 1998), compared with 10 percentage points in the SS and SSLD models.

At first glance, the milder Sudden Stops of the SSLD model in terms of consumption and prices seem inconsistent with the observations that the current account-output ratio is about the same in both models at $t=0$ and the SSLD model always operates with higher debt. These contrasting results are actually critical, because they illustrate the quantitative relevance of the effects of the intermediation externality operating via the changes in ex-ante and ex-post interest rates under liability dollarization. The former, in particular, turns out to be very significant. To show this, use the resource constraint of tradables of the SS and SSLD models to express the difference in tradables consumption $\left(\Delta c_{t}^{T} \equiv c_{t}^{T, S S L D}-c_{t}^{T, S S}\right)$ as follows:

$$
\begin{equation*}
\Delta c^{T}=\Delta y_{t}^{T}+\Delta p_{t}^{c} b_{t}^{c}+\Delta q_{t}^{c} p_{t}^{c} b_{t+1}^{c} \tag{51}
\end{equation*}
$$

where $\Delta y_{t}^{T} \equiv y_{t}^{T, S S L D}-y_{t}^{T, S S}$ is the gap in the realizations of $y^{T}, \Delta p_{t}^{c} b_{t}^{c} \equiv p_{t}^{c} b_{t}^{c}-b_{t}$ is the gap in the repayment value of outstanding debt, and $\Delta q_{t}^{c} p_{t}^{c} b_{t+1}^{c} \equiv q_{t}^{c} p_{t}^{c} b_{t+1}^{c}-q^{*} b_{t+1}$ is the gap in the resources generated by newly issued debt.

At $t=0$, when Sudden Stops occur, $\Delta c_{0}^{T}=0.2$, and this is accounted for by $\Delta y_{0}^{T}=0.021, \Delta p_{0}^{c} b_{0}^{c}=$ $0.034, \Delta q_{0}^{c} p_{0}^{c} b_{1}^{c}=0.149$. Hence, the large increase in the resources generated by new debt that follows from the higher ex-ante bond prices accounts for 1,490 basis points (nearly three-quarters) out of the $20 \%$ higher tradables consumption in the SSLD v. the SS model. The reduction in the burden of debt repayment due to the real exchange rate collapse accounts for 340 basis point, and the higher income with which Sudden Stops occur on average in the SSLD economy (since it carries higher debt levels in units of tradables) accounts for 210 basis points. ${ }^{23}$ Moreover, since the collateral constraint binds at $t=0$, it follows that $\Delta q_{0}^{c} p_{0}^{c} b_{1}^{c}=\kappa\left(\Delta y_{0}^{T}+\bar{y}^{N} \Delta p_{0}^{N}\right)$, where $\Delta p_{0}^{N}=0.142$. Thus, in Sudden Stop states, the effect of the increased resources generated by new debt has an equivalent interpretation in terms of enhanced borrowing capacity: Sudden Stops are weaker in the SSLD model because pledgeable collateral is significantly higher since the nontradables price falls much less and $y^{T}$ is higher. Note that by itself the increase in $q_{0}^{c}$ tightens the collateral constraint, but this increase results from the same forces that are pushing $p_{0}^{N}$ sharply lower, and the latter effect dominates. Of the 0.2 gap in tradables consumption in a Sudden Stop, 0.142 is due to the nontradables price gap, 0.027 to the income gap, and again 0.03 is due to the reduced burden of debt repayment.

### 4.3 Optimal, Time-Consistent Debt Taxes \& Capital Controls

We now examine the key properties of optimal, time-consistent policies. We start by comparing the DE of the SSLD and SS models with the equilibrium of the conditionally efficient social planner (SP). The SP solution is computed using a value function iteration algorithm described in the Appendix. ${ }^{24}$ We then use the planner's allocations together with equations (47) and (49) to construct the optimal schedules of $\tau, \theta$.

[^18]Table 2: Long-run Moments: Decentralized Economies \& Social Planner

| Long-run Moments | $(1)$ | $(2)$ | $(3)$ |
| :--- | :---: | :---: | :---: |
|  | SS | SSLD | SSLD - SP |
| Average $\left(p^{c} b^{c} / Y\right) \%$ | -27.16 | -29.41 | -22.57 |
| Average TB/Y Ratio | 1.22 | 1.12 | 0.80 |
| Welfare gain $1 \%$ | $\mathrm{n} / \mathrm{a}$ | 0.26 | 0.54 |
| Prob. of Sudden Stops $2 \%$ | 4.76 | 3.83 | 0.00 |
| Prob $\left(\mu_{t}>0\right) \%$ | 9.30 | 35.38 | 22.96 |
| Prob of MP tax region \% | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | 49.7 |
| Median Debt Tax Rate $\tau \%$ | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | 5.79 |
| Median Capital Control Rate $\theta \%$ | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | -12.78 |
| Average $c$ | 0.989 | 0.989 | 1.024 |
| Average fall in $c$ in Sudden Stops ${ }^{3}$ | -12.73 | -4.60 | $\mathrm{n} / \mathrm{a}$ |

[^19]Table 2 compares key statistics of the DE of the SS and SSLD models v. the SP solution of the SSLD model. The mean debt ratio of the SSLD model exceeds that of the SS model (as explained earlier) but that of the SP for the SSLD economy is lower than both of them (the ratio of bonds to output is -22.6 percent v. -27.2 and -29.4 percent in the SS and SSLD economies respectively). Hence, the combined effect of the macroprudential and intermediation externalities results in overborrowing on average (by about 7 percentage points of GDP) in the unregulated SSLD economy compared with the socially optimal allocations of the conditionally-efficient SP. The lower average debt ratio implies that under the SP's optimal policies the economy attains
higher average consumption and requires a lower average trade surplus. In addition, the optimal policies deliver a non-trivial average welfare gain of $0.54 \%$ and reduce the probability of Sudden Stops to virtually zero, compared with Sudden Stop probabilities of 4.8 and 3.8 percent in the SSLD and SS economies respectively.

The probability of the constraint being binding is nearly 4 times higher in the SSLD than in the SS economy, which is consistent with higher borrowing under liability dollarization, but Sudden Stops have a lower probability because in several of the states in which $\mu_{t}>0$ in the SSLD economy the current account reversal is not large enough to classify as a Sudden Stop, reflecting again the effects of the intermediation externality that weaken the magnitude of Sudden Stops. Still, the SP regulates the economy so as to reduce the frequency of the constraint being binding in the SSLD economy, from 35.4 to 23 percent. The probability of being in the region in which the macroprudential debt tax is used (i.e. where $\mathbb{E}_{t}\left[\mu_{t+1}\right]>0$ ) is close to $50 \%$, which implies that macroprudential debt taxes are used nearly half the time even if crisis have a near-zero probability of occurrence. The median debt tax rate is nearly $5.8 \%$, while the capital controls rate is actually a subsidy of about $12.8 \%$. Hence, while on average domestic debt is taxed to contain the macroprudential externality and the pro-borrowing incentives of the intermediation externality, capital inflows are subsidized in order to maintain ex-ante real interest rates at the values required to maintain the credibility of the optimal policy.

Table 2 also shows that welfare is higher in the SSLD than in the SS economy, by $0.26 \%$ in terms of a permanent compensating variation in consumption. Notice that the long-run averages of consumption are about the same but the mean drop in consumption in Sudden Stop states is nearly three times higher in the SS economy. Hence, the higher welfare of the SSLD economy is largely due to the smaller and less frequent financial crises than in the SS economy. These results indicate that dollarizing private debt contracts is welfare-reducing when collateral values still hinge largely on domestic relative prices.

The quantitative appeal of the optimal policy reflected in the comparison in Table 2 hides the fact that, as in previous quantitative studies of optimal financial policy in Fisherian models (e.g. Bianchi and Mendoza (2010), Bianchi et al. (2016), Hernández and Mendoza (2017)), the optimal
tax schedules are complex, displaying significant, nonlinear variation over time and across states of nature (i.e. depending on the realization of income and the amount of outstanding debt). Moreover, as those studies also showed, it is not clear that ad-hoc simple rules can attain outcomes similar to those of the optimal policies and in fact they can make the economy worse off than in the absence of regulation! These drawbacks highlight the importance of conducting a detailed quantitative comparison of simple policy rules, which we proceed to do next.

### 4.4 Simple Financial Policy Rules

We study now the effectiveness of simple rules setting $\tau_{t}$ and $\theta_{t}$ by examining their implications for the amount of debt in the economy, the likelihood and severity of Sudden Stops, and social welfare. We construct an algorithm that searches for the welfare-maximizing configuration of each of the following three policy regimes: First, constant values for $\tau$ and $\theta$. Second, a "debt tax Taylor rule," which targets credit by setting $\tau_{t}$ using an isoelastic function of the ratio of bond holdings to a policy target, keeping $\theta_{t}$ constant at the welfare-maximizing value under constant policy rates. Third a "capital controls Taylor rule", which sets $\theta_{t}$ as an isoelastic function of the real exchange rate $\left(p_{t}^{c}\right)$ relative to a target $\bar{p}_{t}^{c}$, keeping $\tau_{t}$ constant at the welfare-maximizing value under constant policies. The intuition is that this rule uses capital controls to manage fluctuations in ex-post and ex-ante real interest rates, which in this economy is akin to targeting the real exchange rate or the relative price of aggregate consumption.

When solving decentralized equilibria with simple policies, we assume that the revenue (cost) generated by positive (negative) values of $\theta_{t}$ is rebated (charged) to private agents as part of their lump-sum transfers (taxes) $T_{t}$. We do this because, if they are passed on to intermediaries, the frictionless formulation of financial intermediation that we adopted renders changes in $\theta$ equivalent to parametric increases in $q^{*}$, and hence subsidizing capital inflows is equivalent to lowering the world interest rate arbitrarily, making very large subsidies optimal. ${ }^{25}$

[^20]Under the above assumption, transfers to private agents are given by $T_{t}=-\tau_{t} p_{t}^{c} b_{t}^{c}-\theta q_{t}^{c} p_{t}^{c} b_{t+1}^{c}$, which together with the agents' budget constraint (20) and the pricing condition (19) yields the same resource constraint for tradables as in the equilibrium without policies (13), thus removing the income effects induced by $\tau$ and $\theta$. Hence, in the equilibrium conditions of the model with simple policies, the two taxes appear only in the Euler equation for bonds forming the wedge that defines $\tau^{e f}$, and in addition $\theta$ appears in the collateral constraint as a term that effectively increases the fraction of income pledgeable as collateral, because it lowers $p_{t}^{c}$ so that the collateral constraint becomes $q^{*} \mathbb{E}\left(p^{c \prime}\right) b^{c^{\prime}} \geq-\kappa(1+\theta)\left(y^{T}+p^{N} y^{N}\right)$. This is important because it implies that, in the absence of the collateral constraint, allocating the budgetary impact of the capital controls to private agents yields again the result that only $\tau^{e f}$ matters (by distorting the intertemporal Euler equation), and decomposing it into $\tau$ and $\theta$ is irrelevant. If the collateral constraint is present this may no longer hold at equilibrium, because $\theta$ has an effect separate from $\tau$ when the constraint binds: It increases (reduces) borrowing capacity as $\theta$ rises (falls). Hence, with simple policies, the model justifies capital controls as a policy specifically aimed at targeting capital inflows only in the presence of credit constraints.

## a) Constant policies regime

In this regime, $\tau_{t}=\tau$ and $\theta_{t}=\theta$ at all times. The values that each instrument can take are defined in discrete grids. We then solve the regulated decentralized equilibrium of the SSLD model for each available pair $(\tau, \theta)$ and search for the welfare-maximizing pair. Figure 6 shows three plots that illustrate the welfare effects of a set of $(\tau, \theta)$ pairs. Panel (a) shows the welfare effects of varying $\tau$ in the $[0,0.06]$ interval for $\theta=[-0.02,-0.01,0.005,0.01,0.02]$ (where 0.005 is the value of $\theta$ that maximizes welfare). Panel (b) shows the welfare effects of varying $\theta$ in the $[-0.03,0.06]$ interval for $\tau=[0.01,0.02,0.03,0.04]$ (where 0.02 is the value of $\tau$ that maximizes welfare). Panel (c) uses the same data of Panel (a), but plotted as a function of the value of $\tau^{e f}$ corresponding to each $(\tau, \theta)$ pair.

Figure 6: Welfare Effects of Constant Policies


Note: The circles identify the maximum value of welfare gains and the corresponding maximum $\tau$ and $\theta$ points for maximizing welfare gains across all $(\tau, \theta)$ pairs.

To understand the intuition behind the shape of these plots, keep in mind that without policy intervention the economy is affected by the macroprudential and intermediation externalities. The simple policies can in principle weaken these externalities, but nothing guarantees a welfareincreasing outcome for arbitrary $(\tau, \theta)$ pairs. Whether this is the case or not depends on the extent to which the distortions introduced by these policies tackle the externalities v . the costs of these distortions themselves. In turn, the distortions that the constant policies introduce are determined by the following effects. First, there are the two distortions evident from the optimality conditions
mentioned earlier: 1) both higher $\tau$ or higher $\theta$ increase the effective real interest rate in the Euler equation for bonds, thus increasing the marginal cost of borrowing; 2) higher $\theta$ increases borrowing capacity by increasing the effective fraction of income pledgeable as collateral. There are also two precautionary-savings effects that are dynamic implications of the first two effects: 3) the interestrate effect of higher $\theta$ or higher $\tau$ strengthens precautionary savings incentives; and 4) the collateral effect of higher $\theta$ reduces the need for precautionary savings. Finally, there is also a Sudden Stops effect: 5) as a result of the previous effects, changes in $\tau$ and/or $\theta$ affect the frequency and magnitude of financial crises, and these effects are non-monotonic (e.g if $\theta$ is so high that the constraint never binds, or is set at $\theta=-1$ so that no debt is allowed, the Fisherian deflation mechanism dissapears).

When considering the precautionary savings effects, it is worth recalling that the stationary asset demand curve of incomplete-markets models, which plots average bond holdings at different interest rates, is generally concave and has an horizontal asymptote where the interest rate equals the rate of time preference. This has two important implications for the effects of $\theta$ and $\tau$. First, because of the concavity, changes of equal size in $\theta$ or $\tau$ have much stronger effects on the average debt position around a high interest rate than a low one. Second, there is an asymmetry between the two instruments in how they alter the stationary debt position: changing $\tau$ or $\theta$ implies similar movements along the asset demand curve, but changing $\theta$ also alters credit limits and thus shifts the asset demand curve.

Panel (a) of Figure 6 shows that for $\tau<0.02$ welfare rises with $\tau$ and is about the same across the five values of $\theta$. In this region, a higher debt tax is beneficial because, via the effects mentioned above, it reduces the adverse effects of the macroprudential and intermediation externalities. The separate effect of $\theta$ on borrowing capacity does not make much difference, because although the lower values of $\theta$ reduce borrowing capacity, the higher debt taxes are already aiming to reduce debt in the economy. In contrast, as $\tau$ increases above 0.02 , welfare starts to decline for each value of $\theta$, and it is much lower for higher $\theta$. This is because now taxing debt has a rapidly growing distortionary effect on borrowing decisions that exceeds the benefits of weakening the externalities, but lowering $\theta$ offsets it partially because the reduced borrowing capacity strengthens incentives for precautionary savings, making the higher marginal cost of borrowing less distortionary.

For each curve corresponding to a given value of $\theta$ in Panel (a), there is an interval of values of $\tau$ for which welfare is nearly independent of $\tau$. This is an implication of the shape of the welfare curves in Panel (b), which show that, for a given value of $\tau$, there is always a threshold value of $\theta$ below which welfare is only marginally increasing in $\theta$. In this region, the interest rate and borrowing capacity effects of $\theta$ push against each other, with low values of $\theta$ reducing the marginal cost of borrowing and precautionary savings because of the former, but also reducing borrowing capacity and increasing precautionary savings because of the latter. The net result is that welfare rises only slightly with $\theta$. On the other hand, for $\theta$ higher than the threshold value, welfare begins to fall sharply as $\theta$ rises, because now the marginal cost of borrowing is rising too much relative to the costs of the externalities, and the increased borrowing capacity is irrelevant. Note also that for a given $\theta$ in this region, welfare is sharply lower at higher $\tau$, because of the stronger distortionary effect of debt taxes.

Panel (c) of Figure 6 illustrates three important properties of the regime with constant policies. First, there is a region of constant tax pairs for which debt taxes and capital controls are equivalent, and hence only the effective debt tax matters. In particular, when the $\tau, \theta$ values yield $\tau^{e f} \geq 0.038$, a given $\tau^{e f}$ yields the same welfare regardless of the value of $\theta$. This is because at high values of $\tau^{e f}$ incentives to borrow are weakened enough to make the effect of $\theta$ on borrowing capacity irrelevant, and as explained earlier, in the absence of this mechanism the two policy instruments are equivalent. But for $\tau^{e f}<0.038$ the equivalence breaks. For a given $\tau^{e f}$ in this region, welfare is lower at higher values of $\theta$. This is a key result, because it shows that when the two instruments are not equivalent, it is preferable to generate a given $\tau^{e f}$ with a mix that uses (weakly) lower capital controls, because in this region a higher borrowing capacity with a higher $\theta$ makes taxing debt more costly. Second, as in Panel (a), for each value of $\theta$ there is an interval of values of $\tau^{e f}$ that generates roughly similar welfare effects and this interval is wider for lower $\theta$, which is again due to the flat region of welfare effects identified in Panel (b). This result is important because it shows that in this model, if the choice is only over constant taxes and $\theta$ is set relatively low, regulators have more "margin of error" for where to set $\tau$ without reducing welfare sharply. Third, in line with the literature, it is easy for simple policies to produce outcomes that reduce welfare relative to the unregulated competitive equilibrium, by as much as as $0.45 \%$ for $\tau^{e f}=0.09$. Any $(\tau, \theta)$ pair that yields a value of $\tau^{e f}$
above 0.045 is worst than leaving the economy unregulated and fully exposed to Sudden Stops, and this is true regardless of the value of $\theta$. Considering that, as noted below, even the best choice of constant policies yields small welfare gains and modest declines in the frequency and severity of Sudden Stops, this result highlights again the importance of careful quantitative evaluation of macro-oriented financial regulation.

The best policy mix obtained by maximizing over both instruments yields a welfare gain of $0.1 \%$, which is attained by setting $\tau^{* e f}=0.025$ with $\tau^{*}=0.02$ and $\theta^{*}=0.005$. Unfortunately, this maximum welfare gain is only $1 / 5$ th the size of the one obtained under the optimal policy.

Summing up, the analysis of the regime with constant debt taxes and capital controls yields three key results: (1) If constant policies yield effective debt taxes that are set high enough, capital controls and domestic credit regulation are equivalent, otherwise capital controls are much less desirable than debt taxes; (2) the welfare-maximizing policy mix is in the region where the two instruments are not equivalent, and consists of a domestic debt tax of $2 \%$ and a tax on foreign capital inflows of $0.5 \%$; (3) the best mix of constant policies generates much smaller welfare gains than the optimal policy, and other values of constant policies not too distant from the best mix can result in large welfare losses relative to the unregulated competitive equilibrium.

## b) Regimes with Taylor rules for debt taxes $\S$ capital controls

We study now Taylor-rule-like rules that set the values of debt taxes or capital controls. In the case of the debt tax, we use a rule similar to the one studied by Bianchi and Mendoza (2017):

$$
\begin{equation*}
\tau_{T}=\max \left\{\left(1+\tau^{*}\right) \cdot\left(\frac{b_{t}^{c}}{\overline{b^{c}}}\right)^{\phi_{T}}-1,0\right\} \tag{52}
\end{equation*}
$$

where $\tau^{*}$ is the best debt tax under constant taxes, $\bar{b}^{c}$ is a credit target that the financial authority sets, and $\phi_{T}$ is the elasticity of the tax to deviations of credit with respect to its target. The value of $\theta_{t}$ is kept constant at its best value under constant policies. This rule includes a "zero lower bound" to incorporate the theoretical result that the macroprudential externality supports only non-negative taxes. ${ }^{26}$ We searched numerically for the pair $\left(\phi_{T}, \bar{b}^{c}\right)$ that maximizes welfare, and

[^21]obtained $\phi_{T}=2.75$ and $\bar{b}^{c}=-0.285$ with an associated welfare gain of $0.119 \%$ relative to the unregulated decentralized equilibrium. The debt target is only about one percentage point lower than the unregulated long-run average (-0.294), but the elasticity implies that the debt tax increases by 275 basis points when the debt rises by 100 basis points above its target. Hence, the debt tax changes rapidly as the debt deviates from its target, with the caveat that the tax is completely lifted when credit is $1 \%$ or more below its target.

The Taylor rule for capital controls has the following functional form:

$$
\begin{equation*}
\theta_{T}=\left(1+\theta^{*}\right) \cdot\left(\frac{p_{t}^{c}}{\bar{p}^{c}}\right)^{\phi_{C}}-1 \tag{53}
\end{equation*}
$$

where $\theta^{*}$ is the optimal capital control rate under constant policies and $\bar{p}^{c}$ is a real-exchangerate target. In this case, capital controls respond to deviations of the real exchange rate from its target with an elasticity $\phi_{C}$. If this elasticity is negative (positive), capital controls are lowered (increased) when the real exchange rate is above its target, which lowers the effective tax on debt but also reduces borrowing capacity. Hence, in Sudden Stop events, in which the real exchange rate collapses, this rule tightens capital controls to reduce the desire to borrow (i.e. reduce the shadow value of the borrowing constraint) and enhances borrowing capacity by increasing effective pledgeable collateral.

We searched numerically for the values of $\phi_{C}$ and $\bar{p}^{c}$ that maximize welfare, and obtained $\phi_{C}=-0.51$ and $\bar{p}^{c}=3.17$, with an associated welfare gain of $0.14 \%$ relative to the unregulated decentralized equilibrium. The real-exchange-rate target is a notch higher than the average value of $p^{c}$ in the unregulated economy, but the elasticity parameter implies that if the real exchange rate appreciates by $1 \%$ above target, $\theta$ is reduced by 51 basis points. The improvement in welfare is a notch higher than with constant taxes and the Taylor rule on debt taxes, but still markedly lower than under the optimal policy.

## c) Comparison of policy regimes

We close this Section with a summary comparison of the three simple policy regimes we studied. Figure 7 compares stochastic steady states by showing the long-run distributions of bond holdings
under each policy regime and in the unregulated equilibrium.
Figure 7: Long-Run Distributions of Bond Holdings


This Figure shows that the constant-taxes and debt-tax Taylor rule regimes shift the distribution of bond holdings to the right relative to the unregulated equilibrium, while the capital controlsTaylor rule partially shows a similar pattern but also shows a mass of probability at large debt positions. Thus, the first two regimes share the property of the optimal policies of aiming to tackle "overborrowing." In fact, the constant taxes and debt-tax Taylor rule regimes rule out completely debt positions larger than $98 \%$ and $95 \%$ of average tradable income respectively, while debt ratios as high as $102 \%$ are possible in the absence of policy intervention. In contrast, the advantage of higher $\theta$ supporting higher borrowing capacity under the capital-controls Taylor rule, results in the probability mass at debt ratios in the 100 to $101.5 \%$ interval in the stochastic steady state of this regime.

Table 3 presents key statistics to compare the effectiveness of the policy regimes. The shifts in the long-run distribution of bond holdings result in reductions in the long-run average debt ratios from $-29.4 \%$ in the unregulated economy to $-29.1 \%$ under constant policies, $-28.2 \%$ with the debttax Taylor rule, and $-22.6 \%$ with the optimal policies. In contrast, the capital-controls Taylor rule sustains a higher mean debt ratio of $-30.5 \%$. Still, these differences in debt ratios translate into
negligible differences in average consumption, because interest rates are in the $3-5 \%$ range so debt service differences are very small across regimes.

Given that mean consumption levels are about the same, the differences in welfare across simple policy regimes and the unregulated equilibrium are largely influenced by differences in the frequency and magnitude of Sudden Stops, and in the frequency with which the collateral constraint binds (even without a Sudden Stop). With constant policies, the welfare gain is all due to the reduction in the Sudden Stops probability from $3.83 \%$ in the unregulated DE to $3.23 \%$ with constant policies, because in fact the drop in consumption in a Sudden Stop is slightly larger. The debt-tax Taylor rule reduces both the probability of crises and their magnitude (the former falls to $2.76 \%$ and the latter us $-4.48 \%$ v. $-4.6 \%$ in the unregulated DE). The capital-controls Taylor rule results in a significantly smaller consumption decline ( $-2.1 \%$ ) but a higher frequency of crises (3.55\%), which yields a slightly higher welfare gain than the other simple rules. This is possible because this regime combines a higher frequency of the collateral constraint being binding with higher $\theta$ values that make borrowing less desirable and increase borrowing capacity. Notice also that capital controls in this model are akin to a dual policy consisting of time-varying debt taxes and regulatory debt-toincome limits. In terms of the frequency of crises, the debt-tax Taylor rule is the most effective of the simple rules, but in terms of welfare and the size of consumption drops during crises the capital controls Taylor rule is the most effective. In contrast with the simple rules, the welfare gain under the optimal policy is partially due to higher mean consumption, by about $3.5 \%$, than in the other regimes and in the unregulated equilibrium, and this is due in turn to the significantly smaller debt ratio. In addition, welfare is higher because Sudden Stops vanish from the economy with optimal policies in the long run. All three simple policy regimes are far from achieving similar outcomes in terms of welfare gains and reduced frequency of Sudden Stops.

The probability of the collateral constraint being binding reflects a similar pattern, it falls slightly under constant policies and significantly more with the debt-tax Taylor rule, but in this instance the probability of a binding collateral constraint in the latter is lower even than under the optimal policies. Hence, under optimal policies the collateral constraint binds more often, but when it binds the current account reversals are much smaller, and so much so that they do not qualify as Sudden Stops.

The values of policy instruments differ significantly across the various regimes. The $2 \%$ fixed debt tax under constant policies is lower than the median of both the debt-tax Taylor rule and the optimal policies, and the median debt tax of the latter is significantly larger than with the debt-tax Taylor rule ( $5.8 \%$ v. $3.6 \%$ ). The $0.5 \%$ fixed tax on capital inflows under constant policies is lower than the $1.73 \%$ median $\theta$ under the capital-controls Tax rule, but both are much higher than the subsidy of nearly $13 \%$ under optimal policies. Hence, the need to set $\theta$ as required to sustain the same ex-ante real interest rate function of the unregulated economy in order to ensure the time-consistency of the optimal policies, combined with the more flexible (state-contingent) environment to take advantage of trading off higher $\tau$ for lower $\theta$ to enhance borrowing capacity, results in optimal policies that generally operate with higher values of $\tau$ and much lower values of $\theta$, and in fact so low that subsidizing capital inflows is optimal. Thus, combining the results of the

Table 3: Effectiveness of Alternative Policy Regimes

| Long-run Moments $^{1}$ | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | DE | CT | TRT | TRCC | SP |
| Average $\left(P^{c} b^{c} / Y\right) \%$ | -29.41 | -29.07 | -28.18 | -30.49 | -22.57 |
| Welfare Gain ${ }^{2 \%}$ | $\mathrm{n} / \mathrm{a}$ | 0.10 | 0.12 | 0.14 | 0.54 |
| Prob. of Sudden Stops $\%$ | 3.83 | 3.23 | 2.76 | 3.55 | 0.00 |
| Prob $\left(\mu_{t}>0\right) \%$ | 35.38 | 31.84 | 7.15 | 71.08 | 22.87 |
| Median Debt Tax Rate $\tau \%$ | $\mathrm{n} / \mathrm{a}$ | 2.00 | 3.59 | 2.00 | 5.79 |
| Median Capital Control Rate $\theta \%$ | $\mathrm{n} / \mathrm{a}$ | 0.50 | 0.50 | 1.73 | -12.78 |
| Average $c$ | 0.989 | 0.989 | 0.990 | 0.989 | 1.024 |
| Average change of $c$ in Sudden Stops $4 \%$ | -4.60 | -4.87 | -4.48 | -2.06 | $\mathrm{n} / \mathrm{a}$ |

${ }^{1}$ DE denotes the unregulated decentralized economy, CT the economy with constant taxes and capital controls, TRT the economy with the debt-tax Taylor rule, TRCC the economy with the capital-control Taylor rule, and SP the conditionally-efficient planner's problem.
${ }^{2}$ Welfare gains are computed as compensating variations in consumption constant across dates and states that equate welfare in the economies with regulation or the social planner's outcome with that in the unregulated decentralized equilibrium. The welfare gain $W$ at state $\left(b^{c}, y^{T}\right)$ is given by $\left(1+W\left(b^{c}, y^{T}\right)\right)^{1-\sigma} V^{\mathrm{DE}}\left(b^{c}, y^{T}\right)=V^{i}\left(b^{c}, y^{T}\right)$. The long-run average is computed using the ergodic distribution of the unregulated economy.
${ }^{3}$ A Sudden Stop is defined as a period in which the constraint binds and the current account raises by more than two standard deviations in the ergodic distribution of the decentralized economy.
${ }^{4}$ The change in $c$ is not computed for the SP case because no Sudden Stops occur in equilibrium.
various policy regimes yields the more general result indicating that, in this model, liability dollarization favors taxing domestic debt more and capital inflows less, and under the complex regime of the optimal policies, large but carefully managed subsidies on inflows are actually optimal.

Finally, we compare the effectiveness of the policy regimes in term of their implications for Sudden Stop dynamics. Figure 8 compares Sudden Stops in the three simple policy regimes v. the unregulated economy. In order to construct these event windows, we apply the same procedure as in the comparison of the SS and SSLD models.

Figure 8: Sudden Stop Events: Unregulated Economy v. Regimes with Simple Policies


Note: All variables except those measured as output ratios are plotted as percent deviations of their corresponding long-run averages. Variables measured as output ratios are shown as differences relative to the long-run average of the corresponding ratio and expressed in percent.

This Figure shows a striking result: In terms of Sudden Stop dynamics, the regime with the capital-controls Taylor rule yields larger pre-crises expansions, less severe Sudden Stop drops in consumption and the real exchange rate, and smoother post-crisis recoveries. Aggregate consumption,
tradables consumption and the real exchange rate fall significantly less, and the current account reversal is smaller, with the capital-controls Taylor rule. In contrast, the deb-tax Taylor rule and the constant-taxes regime yield consumption and price outcomes that do not differ much from the unregulated equilibrium, although the former does perform better in some features of crisis dynamics (e.g. the reversals in debt and the current account are smaller). The event plots also show that the main reason the capital-controls Taylor rule outperforms the other regimes is because it yields dynamics in current v . expected real exchange rates that virtually stabilize ex-ante debt prices. This minimizes the distortionary effect of the intermediation externality operating through changes in resources that newly issued debt generates because of ex-ante interest rate fluctuations. This in turn allows this regime to maintain debt access stable and face a small current account reversal, which is mainly caused by the fall in total output due to lower $y_{0}^{T}$ and $p_{0}^{N}$. The capital-controls Taylor rule achieves this performance by tightening capital controls sharply when a Sudden Stop hits (from $\theta_{-1}=1 \%$ to $\theta_{0}=7.5 \%$ ), which increases pleadgeable collateral by 650 basis points, while at the same time increasing the effective debt tax from $3 \%$ to $9.5 \%$. This improves performance because during Sudden Stops the benefit of relaxing the collateral constraint is significantly larger than the cost of the distortion on borrowing costs. In contrast, the debt-tax Taylor rule has the ability to discourage borrowing above the credit target, implying that at the moment of a Sudden Stop the economy is in better shape to face low income realizations and a binding collateral constraint, but it does not have the ability to ease access to credit.

In summary, the comparison of the effectiveness of the policy regimes in terms of frequency and magnitude of Sudden Stops and social welfare shows that: (a) the optimal time-consistent policies are very effective, but also very complex, (b) the policy mix favors using debt taxes relatively more than capital controls, with simple rules imposing much larger taxes on domestic debt than on capital inflows, and the optimal policies taxing debt even more and actually subsidizing capital inflows, and (c) it is possible to construct simple policy rules that are welfare-improving, but they are significantly less effective than the optimal policies, and of these rules a capital-controls Taylor rule that targets the real exchange rate performs better, because it enhances borrowing capacity during Sudden Stops by tightening capital controls.

## 5 Conclusions

We proposed a model of Sudden Stops with liability dollarization in which frictionless banks intermediate foreign liabilities in units of world tradable goods into domestic loans denominated in units of aggregate consumption, which is a composite good that combines tradables and nontradables. As in standard Sudden Stops models, a collateral constraint limits debt in units of tradables not to exceed a fraction of the market value of total income in the same units, so that the equilibrium relative price of nontradables enters as a determinant of borrowing capacity. We showed that liability dollarization alters the predictions of standard Sudden Stops models, in which domestic loans are issued in units of tradables, in important ways. Under perfect foresight, Sudden Stops are less severe because real-exchange-rate collapse (i.e. a collapse in the price of aggregate consumption) reduces the burden of repaying outstanding debt. Under uncertainty, however, there are two important additional effects: Real-exchange-rate expectations affect ex-ante real interest rates and hence the resources generated by issuing new debt, and the positive co-movement of consumption and ex-post real interest rates reduces the expected marginal cost of borrowing. These effects add an "intermediation externality" to the macroprudential externality of the standard Sudden Stops models.

We examined optimal policy under commitment and showed that it is time-inconsistent, it tightens access to debt when expectations of real appreciation rise, and it does not require capital controls, because domestic debt taxes and capital controls are equivalent and only their combined effect altering the marginal cost of borrowing matters. Time-inconsistency emerges because during crises the regulator favors higher expected post-crisis prices, which increase ex-ante debt prices and the resources generated by newly issued debt in crisis periods, but in post-crisis periods lower prices are more desirable to reduce the burden of repayment of outstanding debt. In contrast, we showed that the optimal, time-consistent policy of a conditionally-efficient regulator that faces the same pricing function for domestic debt as in the unregulated economy, requires both domestic credit regulation and taxes or subsidies on capital inflows.

A quantitative analysis comparing the liability dollarization model of Sudden Stops with the standard Sudden Stops model based on a widely-used calibration for Argentina produced interesting
results about the effectiveness of the optimal, time-consistent policies and of simple policy rules (constant taxes on debt and capital inflows, and Taylor-like rules driving debt taxes and capital controls). The liability dollarization model generates more debt than the standard model, but it also generates less frequent and less severe Sudden Stops. On the other hand, the Sudden Stops of the liability dollarization model match more closely the empirical regularities of Sudden Stops in terms of the magnitude of reversals in consumption, the real exchange rate and the trade balance.

Regarding the policy analysis, the optimal policies are very effective in that they yield a welfare gain of about $0.5 \%$ and reduce the probability of Sudden Stops to zero, but they are also complex state-contingent and non-linear rules. On the other hand, simple rules optimized to maximize welfare are less effective, and implemented with ad-hoc values can reduce welfare significantly. Simple rules favor larger taxes on domestic debt than on capital inflows, and under optimal policies domestic debt taxes are even larger and subsidies to inflows can be optimal.

An important limitation of this paper is that it abstracted from modeling frictions in financial intermediation, other than liability dollarization. It focused instead on credit frictions affecting private agents, including the collateral constraint and the fact that the non-state-contingent nature of debt contracts together with liability dollarization introduces a wedge by which lenders care about ex-ante domestic real interest rates but borrowers care about their ex-post counterpart. Further research should consider incorporating important additional frictions in financial intermediation, and in particular the possibility of bankruptcy and/or non-neutral bank balance sheet effects as a result of ex-post variations in real returns.

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[^0]:    ${ }^{1}$ Capital Flows and Emerging Market Economies, CGFS Papers No. 33, Bank for International Settlements, January 2009.
    ${ }^{2}$ This framework originated in the seminal articles by Salter (1959) and Swan (1960), and also Díaz-Alejandro

[^1]:    (1965).
    ${ }^{3}$ This setup originates in the work of Mendoza (2002). Studies that explore the models' normative implications, and in particular the implications for macroprudential policy, include Bianchi (2011), Benigno et al. (2016), Korinek (2011), Schmitt-Grohé and Uribe (2017) Bianchi et al. (2016), and Hernández and Mendoza (2017).
    ${ }^{4}$ This implies that the standard SS model of macroprudential policy does not justify the use of capital controls, namely controls intended to discriminate foreign from domestic financial intermediaries, but only the use of financial policies that apply equally to all intermediaries.

[^2]:    ${ }^{5}$ Since we adopt the standard assumption that domestic consumption is a CES aggregate of tradables and nontradables, the relative price of aggregate consumption is a monotonic, increasing function of the relative price of nontradables.

[^3]:    ${ }^{6}$ The price of domestic debt depends on expectations of the (gross) rate of growth of the relative price of the CES composite relative to tradables (i.e. the expected growth rate of the real exchange rate), in a manner akin to the classic Dornbusch (1983) model connecting the domestic real interest rate with the price of nontradables.

[^4]:    ${ }^{7}$ This condition follows from a straightforward optimization problem in which intermediaries maximize the expected present discounted value of their dividends, discounted at the world real interest rate, without any constraints on dividends or liability.

[^5]:    ${ }^{8}$ These assumptions prevent ex-post real-exchange-rate depreciations from causing bank failures. Choi and Cook (2004) study a New Keynesian DSGE model of liability dollarization with costly state verification in which unexpected exchange rate changes affect the external finance premium because of the risk of bank defaults.

[^6]:    ${ }^{9}$ Similarly, keeping $\tilde{b}_{0}^{c}$ and $b_{0}$ unchanged when making parametric changes that affect wealth (for example, temporary or permanent, unanticipated changes in the tradables income stream) results in different equilibria that depend on the changes in initial prices and debt repayment burden.

[^7]:    ${ }^{10}$ The fact that $p^{c \prime}(t) \tilde{b}_{0}^{c}<0$ implies that $m^{S S L D}>m^{S S}$, and the fact that $\omega^{1 / \eta} \tilde{b}_{0}^{c}<b_{0}$ implies that $I^{S S L D}>I^{S S}$.

[^8]:    ${ }^{11}$ The lower marginal cost of borrowing can also be derived by noting that the expected marginal rate of substitution in consumption $\left(u_{T}(t) / \beta \mathbb{E}_{t}\left(u_{T}(t+1)\right)\right.$ in the SSLD model when the collateral constraint does not bind is given by $R_{t+1}^{*}+\frac{\left[\operatorname{Cov}_{t}\left(u_{T}(t+1), \tilde{R}_{t+1}^{c}\right)\right]}{\mathbb{E}_{t}\left(u_{T}(t+1)\right)}$, whereas in the standard SS model it is just $R_{t+1}^{*}$.
    ${ }^{12}$ More generally, the risk-taking incentive is stronger the higher the conditional variability of aggregate consumption, since $u_{T}(),. p^{c}($.$) and c$ are all univariate, monotonic functions of $c^{T}$.

[^9]:    ${ }^{13}$ The last assumption is equivalent to assuming that the planner cannot contract debt directly with foreign lenders in units of tradables, and instead borrows from the same intermediaries as private agents.

[^10]:    ${ }^{14}$ This is because $b_{t}^{c}<0$ implies $-p^{c \prime}(t) b_{t}^{c}>0$ and because the term in square brackets has the same sign as $\lambda_{t-1}-\mu_{t-1}$, which is positive because of condition (27) lagged one period.

[^11]:    ${ }^{15}$ As we explained in Section 2, the only effect of the intermediation externality present under perfect foresight is the debt repayment burden of the exogenous initial debt $p_{0}^{c} b_{0}^{c}$. This can be seen in condition (29) because for $t=0$ there is no matching term $\mathbb{E}_{-1}\left[\lambda_{0}\right]$ to cancel the two terms with $p^{c \prime}(t) b_{t}^{c}$. Hence, the planner has the incentive to increase $p_{0}^{c}$ to reduce the initial debt repayment burden.
    ${ }^{16}$ When $\mu_{t}>0$, conditions (24) and (25) imply that $c_{t}^{T}$ solves the same non-linear equation as in the unregulated competitive equilibrium (i.e. condition (16)).

[^12]:    ${ }^{17}$ For the planner with commitment, $\lambda_{t}>\mu_{t}$ follows from condition (27), and for the conditionally efficient planner $\lambda_{t}>\mu_{t}$ follows from the first-order conditions of the planner's dynamic programming problem under the standard property that $V\left(b^{c}, y^{T}, y^{N}\right)$ is increasing in $b^{c}$, which we verify numerically.

[^13]:    ${ }^{18}$ Note that using (26) we can re-write $\psi(t)$ as $\psi(t)=p^{c \prime}(t)\left[b_{t}^{c}-\frac{q_{t-1}^{*}}{\beta} b_{t}^{c}\left(\lambda_{t-1}-\mu_{t-1}\right)\right]$.

[^14]:    ${ }^{19} \Omega(t+1)=\left(1-\left(1-\frac{\mu_{t+1}}{\lambda_{t+1}}\right) q^{D E^{\prime}}(t+1) b_{t+2}^{c}\right)<1$ for $b_{t+2}^{c}<0$ because $\lambda_{t+1}>\mu_{t+1}$ and $q^{D E}$ is decreasing in $b^{c}$.

[^15]:    ${ }^{20}$ The credit limits for bonds in units of tradables are given by $-\kappa\left(y^{T}+p^{N}\left(b^{c}, y^{T}\right) y^{N}\right) / q^{*}$ in the SS model and by $-\kappa\left(y^{T}+p^{N}\left(b^{c}, y^{T}\right) y^{N}\right) / q^{c}\left(b^{c}, y^{T}\right)$ in the SSLD model.

[^16]:    ${ }^{21}$ Given the definition of $C A$, current account surpluses (deficits) are measured in Figure 2 by how far above (below) the 45 degree line are the debt decision rules.

[^17]:    ${ }^{22}$ Output in units of tradables is given by $Y_{t}=y_{t}^{T}+p_{t}^{N} y^{N}$.

[^18]:    ${ }^{23}$ Since the long-run average of $c^{T}$ is about the same in both models, the decomposition is nearly the same in terms of deviations from the mean, and hence the result shown in Plot b) that $c^{T}$ in the SSLD model is 20 percentage points higher than in the SS model.
    ${ }^{24}$ It is worth noting that, for sufficiently high $\eta$ and sufficiently low $b^{\prime c}$, the planner's value function can become convex, as allocations in which high debt, high consumption and high prices sustain each other become feasible and produce a non-convexity in the resource constraint. As in optimal growth models with nonconvexities (see Le Van and Dana (2003)), this can generate discontinuities in decision rules and dynamics with "debt traps", in which debt decision rules converge to the lower bound of the state space but sustain high consumption levels.

[^19]:    ${ }^{1}$ Welfare gains are computed as compensating variations in consumption constant across dates and states that equate welfare in the DE and SP cases of the SSLD model. The welfare gain $W$ at state $\left(b^{c}, y^{T}\right)$ is given by $(1+$ $\left.W\left(b^{c}, y^{T}\right)\right)^{1-\sigma} V^{\mathrm{DE}}\left(b^{c}, y^{T}\right)=V^{\mathrm{SP}}\left(b^{c}, y^{T}\right)$. The long-run average is computed using the ergodic distribution of the decentralized problem. For the SSLD welfare gain we compare the SS and SSLD models using the same formula described above, but using the ergodic distribution of the SS model to compute the longrun average.
    ${ }^{2}$ A Sudden Stop is defined as a period in which the constraint binds and the current account raises by more than two standard deviations.
    ${ }^{3}$ Average consumption drop in Sudden Stop states in percent of long-run means. Not shown for the SP case because Sudden Stops are a zero probability event.

[^20]:    ${ }^{25}$ If intermediaries pay the lump-sum taxes to finance these subsidies, the taxes cause a harmless fall in dividends, because there is no limit on bank liability and no constraint requiring bank dividends to be positive. Lowering $\theta$ so as to approach -1 would then be optimal, because while the subsidy-adjusted value of collateral $\left(-(1+\theta) \kappa\left(y^{T}+p^{N} y^{N}\right)\right)$ allows only for an infinitesimally small amount of debt, the amount of resources in units of tradables that this debt generates $\left(-\left(q^{*} \mathbb{E}\left(p^{c \prime}\right) /(1+\theta)\right) b^{c \prime}\right)$ is growing infinitely large.

[^21]:    ${ }^{26}$ Solving the case without this zero lower bound yields lower maximum welfare than with it.

